System Unavailability Analysis Based on Window-Observed Recurrent Event Data

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Outline

- Background
- Data Overview (Re-Scaled)
- Statistical Modeling and Inference
  - Nonhomogeneous Poisson Process
  - Multiple Event Types
  - Bath-tub Intensity Function
  - Truncated Lognormal Event Duration
- Unavailability Analysis
- Conclusions and Areas for Future Research

Background

- Many service industries, such as airlines, telecommunication, and railways, require a high level of system availability in order to be competitive.

- Service providers often offer unavailability guarantees to their customers.

- That is, the unavailable percentage during a certain time period (e.g., a year) is guaranteed to be below a threshold that is specified in the service contract.

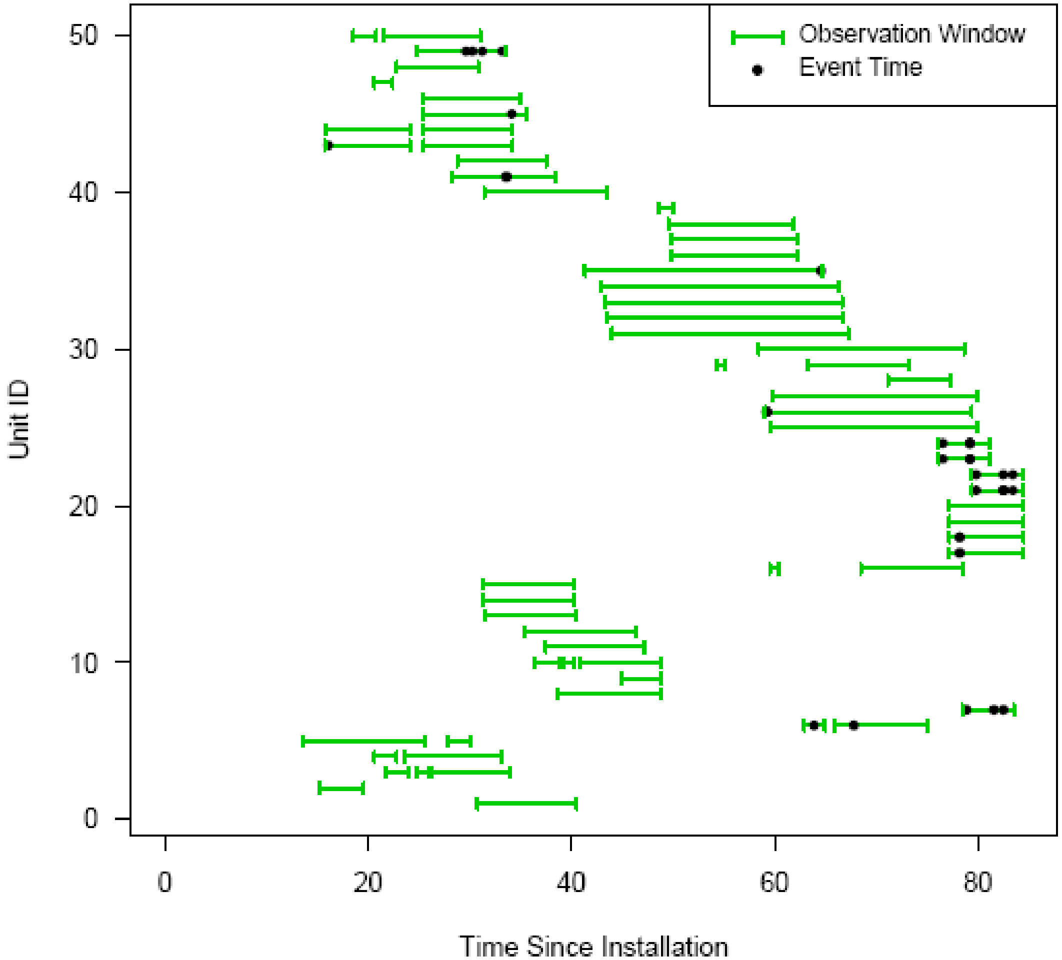
Motivation for System Unavailability Analysis

- The service providers naturally want to offer a low threshold on the unavailability guarantee, to make the business competitive.
- Unfulfillment of a service contract with customers, however, will lead to cost to the service provider.
- Before the service provider decides what to offer, a system unavailability analysis is needed to minimize the risk of unfulfillment.
- Good to be conservative. A conservative metric for system unavailability is desirable.

Application

- Subjects: A fleet of heavy duty industry equipment (System A).
- Goal: Do an unavailability analysis based on historical data.
- System A is a repairable system.
- Good to have: continuous track of all repairs and maintenance during the life of each unit.
- Reality: Only have records for certain time intervals and no information beyond such intervals.

System A Data Overview (Re-Scaled)



- Observation windows
- Event times
- Multiple event types
- Event durations

Analysis Strategy

- Statistics: obtain maximum information from the limited data source.

- Modeling: extend NHPP for repairable system + distribution of event duration

- Unavailability: Define a conservative metric for system unavailability and use simulation to estimate it

Statistical Modeling and Inference

- ❖ Nonhomogeneous Poisson Process
- ❖ Multiple Event Types
- ❖ Bath-tub Intensity Function
- ❖ Truncated Lognormal Event Durations

Nonhomogeneous Poisson Process (NHPP)

- For a repairable system, the NHPP is usually used:
  - Poisson Process has constant intensity
  - With non-constant intensity function
  - The intensity is a function of time

- The intensity function

$$\lambda(t;\boldsymbol{\theta}) = \lim_{\Delta \to 0} \frac{\Pr[N(t+\Delta) - N(t) = 1]}{\Delta}$$

- Cumulative intensity function

$$\Lambda(t;\boldsymbol{\theta}) = \int_0^t \lambda(s;\boldsymbol{\theta})\,ds$$

Power Law Relationship

❖ Event intensity:

$$\lambda_{\mathrm{pl}}(t;\beta,\eta) = \left(\frac{\beta}{\eta}\right)\left(\frac{t}{\eta}\right)^{\beta-1}$$

❖ Flexible:

$$\beta\begin{cases} <1 & \text{: Decreasing intensity (infant mortality)} \\ =1 & \text{: Constant intensity (failures by random)} \\ >1 & \text{: Increasing intensity (wear out failure)} \end{cases}$$

❖ Cumulative event intensity:

$$\Lambda_{\mathrm{pl}}(t;\beta,\eta) = \int_0^t \lambda_{\mathrm{pl}}(s;\beta,\eta)\,ds = \left(\frac{t}{\eta}\right)^{\beta}.$$

Multiple Event Types

❖ There are multiple types of events to cause a unit out of services.

❖ For example, for a car, engine failure, transmission failure, and flat tire are three types of event.

❖ The overall intensity for the system is:

$$\lambda(t;\boldsymbol{\theta}) = \sum_{k=1}^{K} \lambda_k(t;\boldsymbol{\theta})$$

❖ In System-A data set, there are two failure modes, i.e., k=2, and

$$\lambda(t;\boldsymbol{\theta}) = \sum_{k=1}^{2} \lambda_k(t;\boldsymbol{\theta})$$

The Bath-tub Intensity Function

❖ To fit a model with bathtub shape intensity function, we use a mixture of the PL intensity functions.

❖For a repairable system subject to three different process:

- the decreasing intensity $0 < \beta < 1$
- the increasing intensity $\beta > 1$
- the constant intensity $\beta = 1$

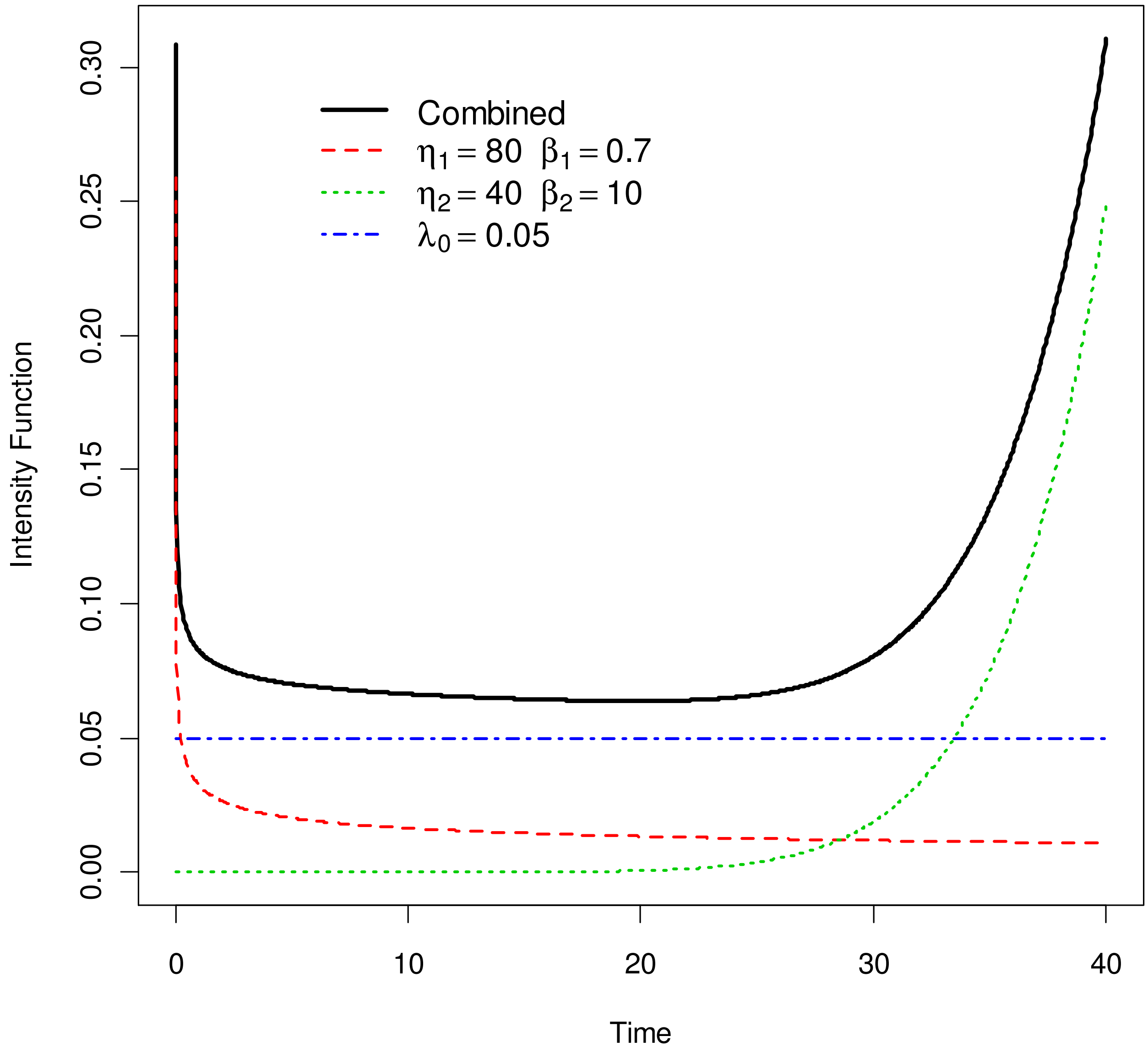
❖ Thus the combined event intensity is:

$$\lambda(t) = \lambda_0 + \frac{\beta_1}{\eta_1}\left(\frac{t}{\eta_1}\right)^{\beta_1 - 1} + \frac{\beta_2}{\eta_2}\left(\frac{t}{\eta_2}\right)^{\beta_2 - 1}$$

❖ The cumulative intensity function is given by:

$$\Lambda(t) = \lambda_0 t + \left(\frac{t}{\eta_1}\right)^{\beta_1} + \left(\frac{t}{\eta_2}\right)^{\beta_2}.$$

Example of Bath-tub Intensity Function



Maximum Likelihood Estimation

❖ Likelihood Function:

$$L(\boldsymbol{\theta} \mid DATA) = \prod_{i=1}^{n} \left\{ \prod_{k=1}^{K} \prod_{j=1}^{n_i} \left[\lambda_k (t_{ij}; \boldsymbol{\theta}_k)^{\mathbf{1}(\delta_{ij}=k)} \right] \times \exp\left(-\sum_{k=1}^{K} \sum_{l=1}^{m_i} [\Lambda_k(\tau_{il}^R; \boldsymbol{\theta}_k) - \Lambda_k(\tau_{il}^L; \boldsymbol{\theta}_k)] \right) \right\}$$

❖ The ML estimates are obtained by finding $\boldsymbol{\theta}$ that maximizes the likelihood function

❖ Local Information Matrix:

$$I_{\hat{\boldsymbol{\theta}}} = \left[-\frac{\partial^2 \log[L(\boldsymbol{\theta} \mid DATA)]}{\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}'} \right] \Bigg|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}}.$$

❖ The large-sample approximate variance–covariance matrix is $\Sigma_{\hat{\boldsymbol{\theta}}} = I_{\hat{\boldsymbol{\theta}}}^{-1}$, if the model assumptions hold well.

Robust Inference

- In reality, there might be some departures from the assumptions of the NHPP.
- The sandwich-type robust variance estimator is used to assure valid statistical inferences.
- The log-likelihood function is $l(\boldsymbol{\theta}) = \sum_{i=1}^{n} l_i(\boldsymbol{\theta})$ where

$$l_i(\boldsymbol{\theta}) = \sum_{k=1}^{K} \sum_{j=1}^{n_i} 1_{(\delta_{ij}=k)} \log\left[\lambda_k(t_{ij}; \boldsymbol{\theta}_k)\right] - \sum_{k=1}^{K} \sum_{l=1}^{m_i} \left[\Lambda_k(\tau_{il}^{R}; \boldsymbol{\theta}_k) - \Lambda_k(\tau_{il}^{L}; \boldsymbol{\theta}_k)\right]$$

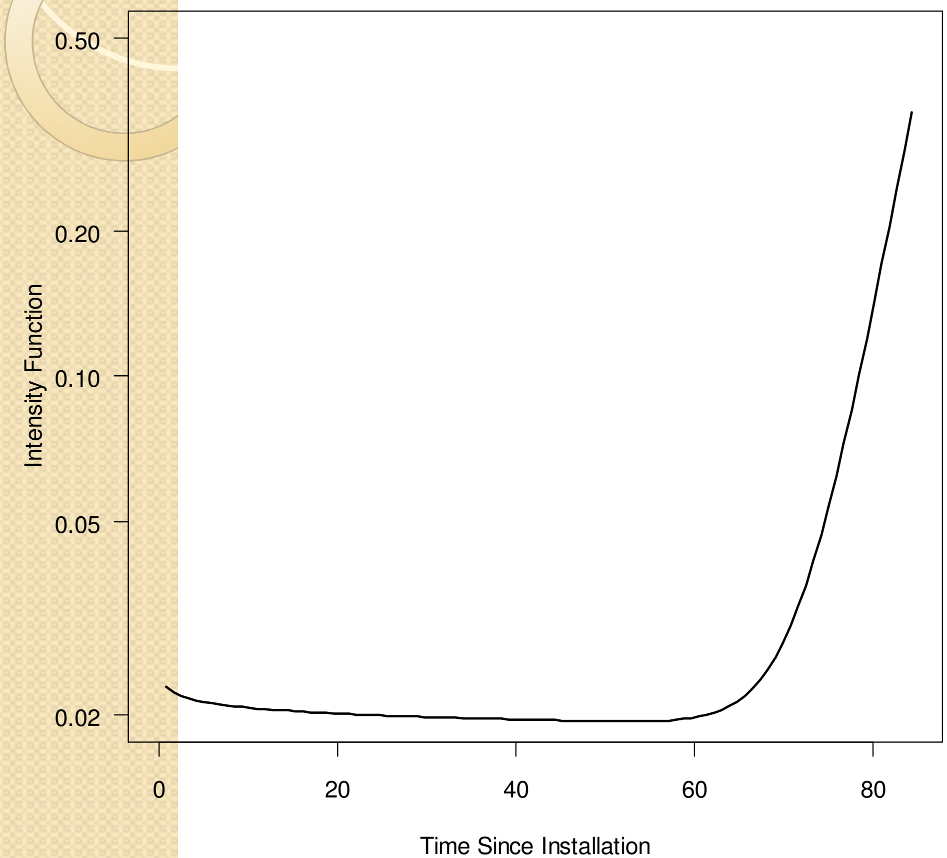
- Let $\mathrm{l}(\boldsymbol{\theta}) = [l_1(\boldsymbol{\theta}), \cdots, l_n(\boldsymbol{\theta})]'$, $U_n(\boldsymbol{\theta}) = \dfrac{\partial \mathrm{l}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}$, $J_n(\boldsymbol{\theta}) = U_n(\boldsymbol{\theta}) U_n(\boldsymbol{\theta})'$.
- The asymptotic theory provides

$$\sqrt{n}(\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta}^*) \rightarrow \mathrm{N}\left[0, I(\boldsymbol{\theta}^*)^{-1} J(\boldsymbol{\theta}^*) I(\boldsymbol{\theta}^*)^{-1}\right], \text{ as } n \rightarrow \infty,$$

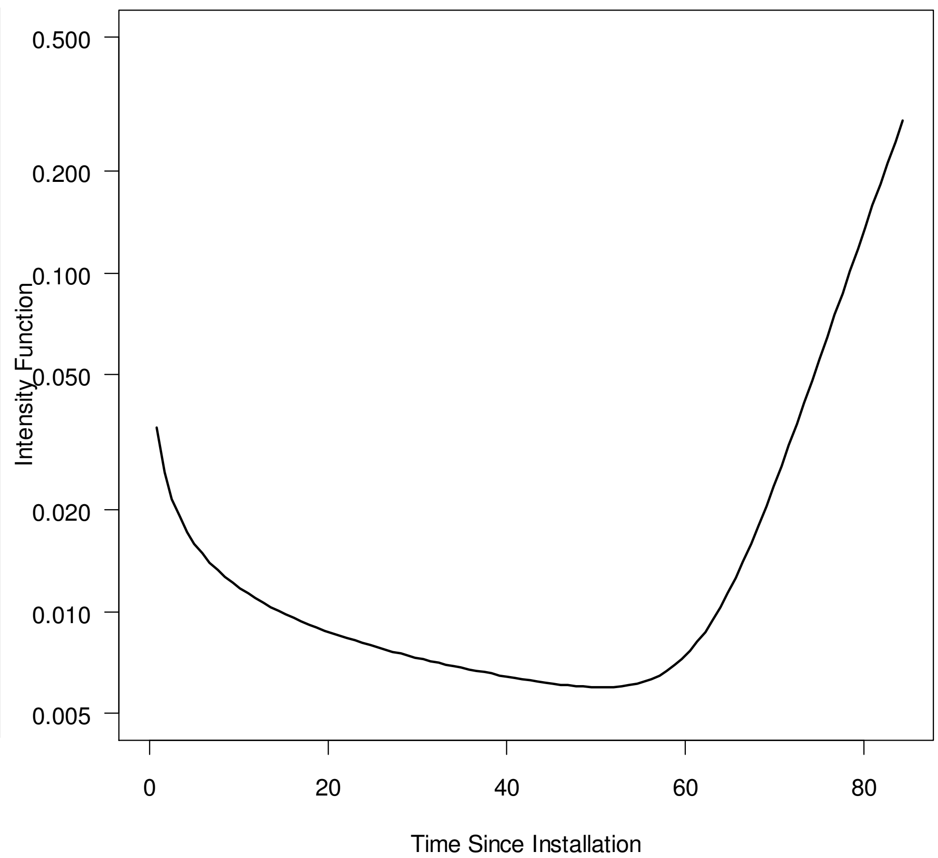
where

$$I(\boldsymbol{\theta}^*) = \lim_{n\to\infty} \frac{1}{n} I_n(\boldsymbol{\theta}^*), \quad \text{and} \quad J(\boldsymbol{\theta}^*) = \lim_{n\to\infty} \frac{1}{n} J_n(\boldsymbol{\theta}^*$$

Estimated Intensity Functions for System A

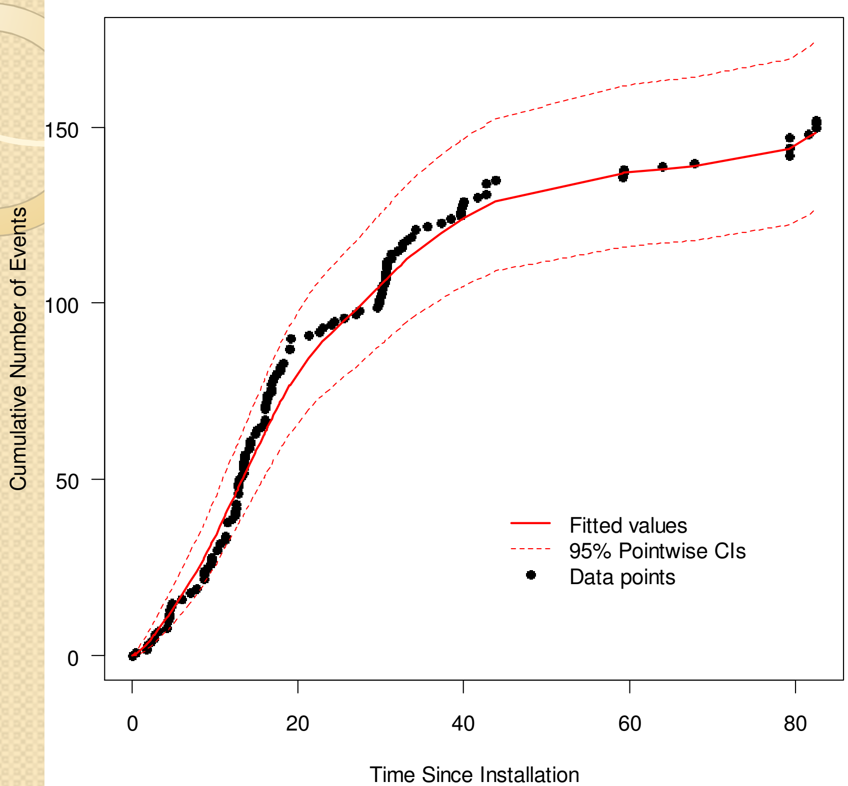


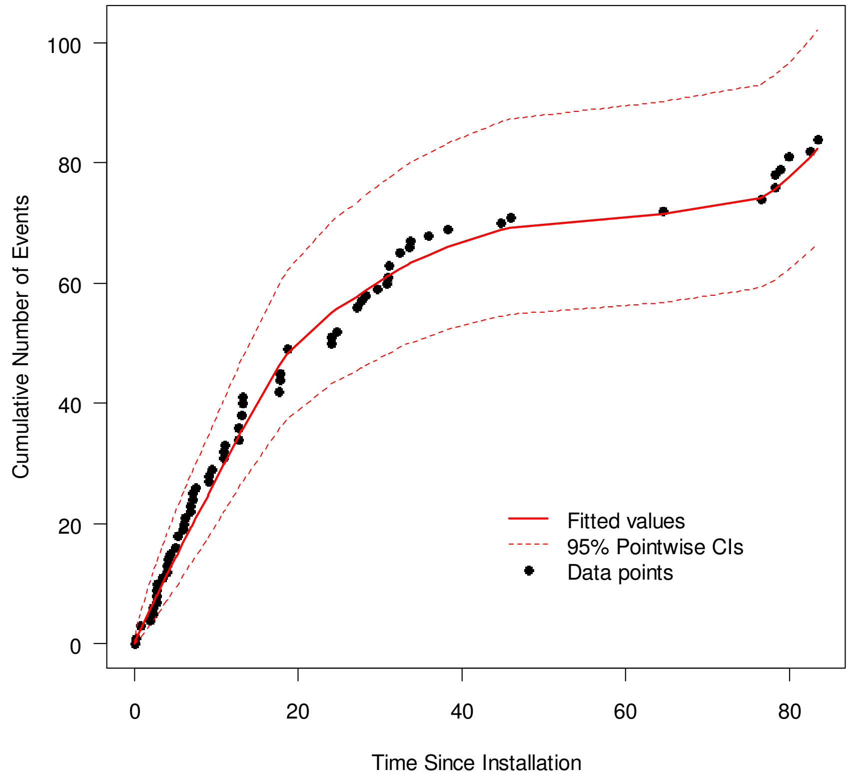
FM1



FM2

Access Model Fitting

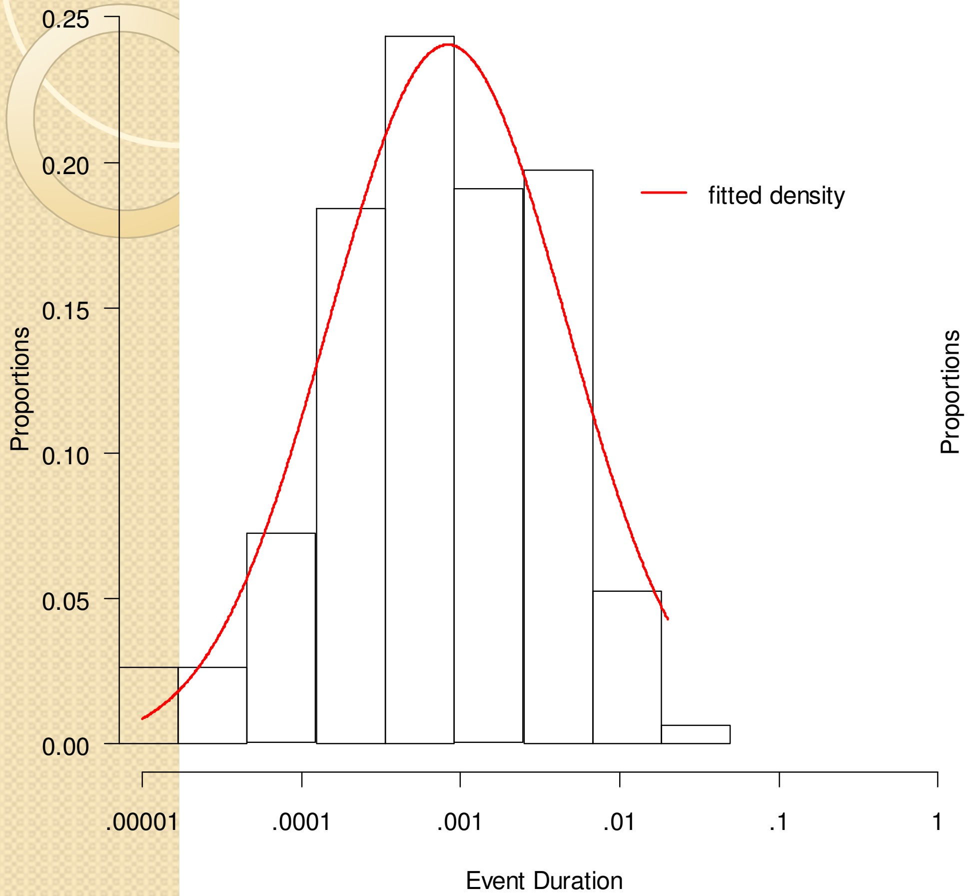


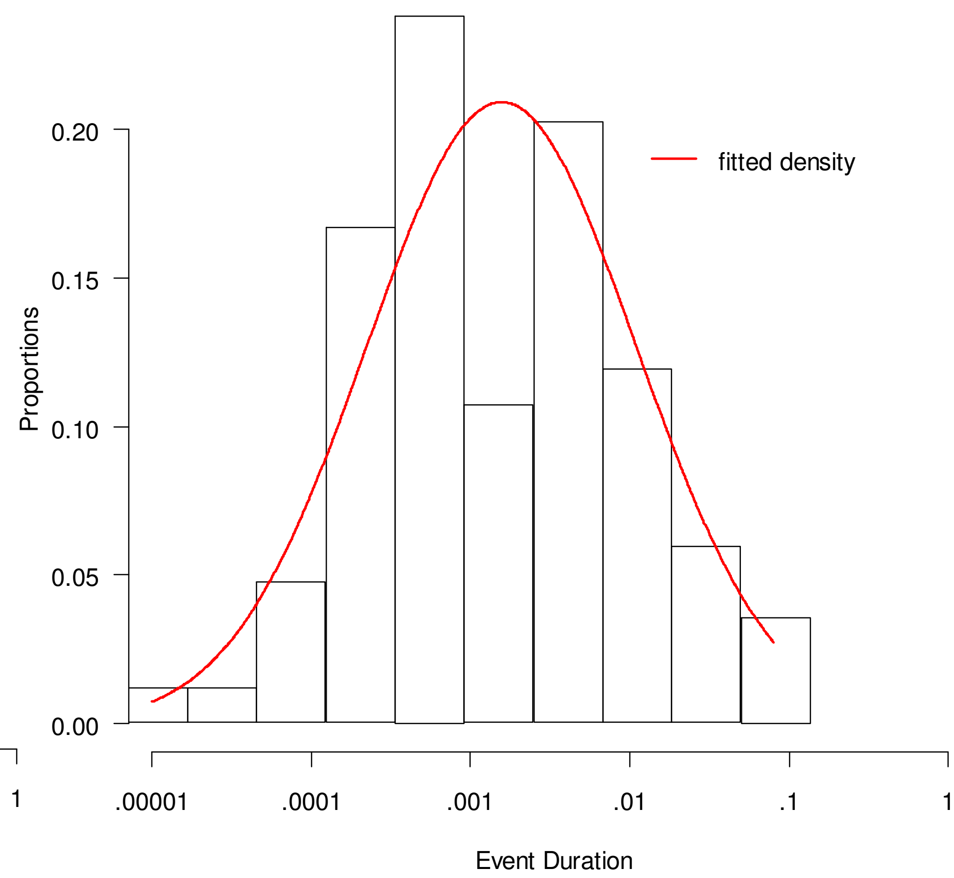


❖ Observation-window-adjusted observed versus estimated expected cumulative number of events

$$E_k(t;\boldsymbol{\theta}_k) = \sum_{i=1}^{n}\sum_{l=1}^{m_i} \mathbf{1}_{\left(\tau_{il}^{L} \le t\right)} \left[\Lambda_k(\min\{t, \tau_{il}^{R}\};\boldsymbol{\theta}_k) - \Lambda_k(\tau_{il}^{L};\boldsymbol{\theta}_k)\right].$$

Event Duration Modeling





❖ Truncated Log-Normal Distribution

Truncated Lognormal Distribution

❖ There is often an upper limit for the event duration

❖ The event durations for FM1 and FM2 of System A are typically less than 0.02 TU and 0.08 TU

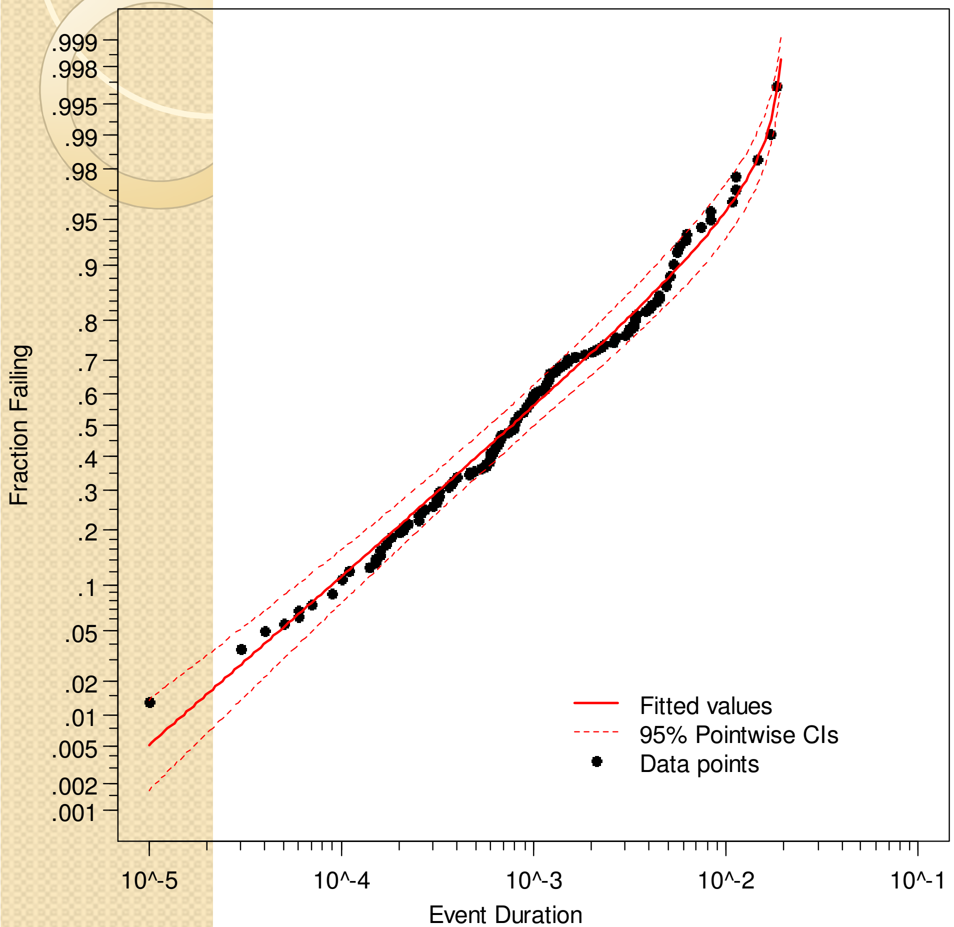
❖ Truncated lognormal is used to model the event duration

❖ The cdf and pdf for truncated lognormal distribution are

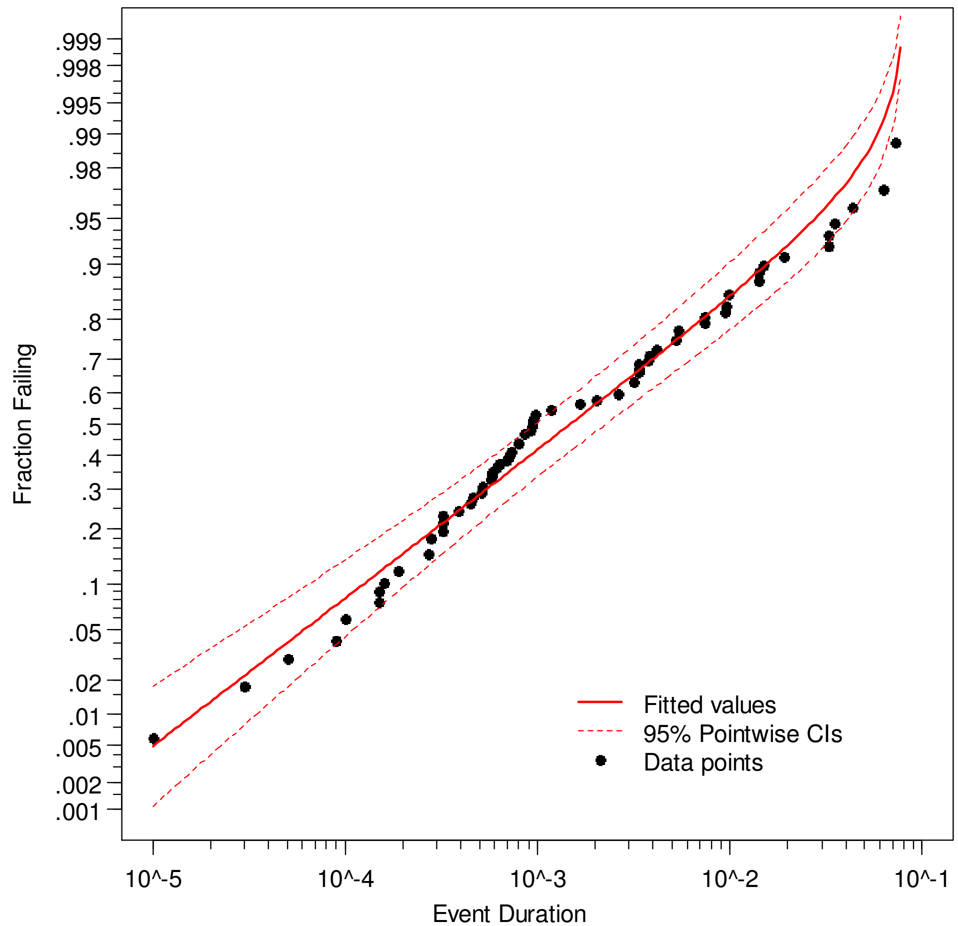
$$F_{\text{TLN}}(x;\psi) = \frac{F(x;\psi)}{F(x_R;\psi)} \quad \text{and} \quad f_{\text{TLN}}(x;\psi) = \frac{f(x;\psi)}{F(x_R;\psi)}, \quad 0 < x \le x_R,$$

❖ Parameters are estimated by ML method and sandwich type variance estimator is used for inference.

Assess Model Fitting
Truncated Lognormal on Lognormal Probability Plot



FM1



FM2

Unavailability Analysis

Unavailable Duration

❖ The unavailable duration within a certain time interval is of interest

❖ The total unavailable time in the TU interval ω is

$$Y_\omega = \sum_{k=1}^{K} \mathbf{1}_{(Z_{k\omega}>0)} \sum_{l=1}^{Z_{k\omega}} X_{kl}$$

❖ $Z_{k\omega} = N_k(\omega) - N_k(\omega-1)$ is the number of events of type *k* which occur during interval $(\omega-1,\ \omega]$

❖ $Z_{k\omega}$ follows the Poisson distribution with intensity parameter $\xi_{k\omega} = \Lambda_k(\omega) - \Lambda_k(\omega-1)$.

❖ X_k is the duration for an event with type *k* .

Unavailability Metric

❖ The traditional metric for system unavailability is

$$u_m(\omega) = \frac{\mathbf{E}(\text{downtime})}{\text{Length of the time interval}} = \frac{\mathbf{E}(Y_\omega)}{1} = \mathbf{E}(Y_\omega).$$

❖ Only considers the mean of unavailable time

❖ The distributional variability is not considered

❖ A conservative metric is needed to quantify the variability in the system unavailable duration

❖ Important to consider the possibility of extreme events

❖ A *Quantile*-unavailability (*Q*-unavailability) metric is defined as

$$u_p(\omega) = F_\omega^{-1}(p)$$

$F_\omega(y)$ is the cdf of Y_ω

❖ It is the *p* quantile of the distribution, e.g., *p*=0.95, 0.99

Unavailability Metric Estimation Algorithm

1. Simulate $Z^*_{k\omega}$ from Poisson$(\hat{\xi}_{k\omega})$, $k = 1, \ldots, K$.
2. If $\sum_{k=1}^{K} Z^*_{k\omega} > 0$, go to step 3.

   If $\sum_{k=1}^{K} Z^*_{k\omega} = 0$, set $Y_\omega^{\ *} = 0$ and go to step 5.
3. For those $Z^*_{k\omega} > 0$, simulate $Z^*_{k\omega}$ number of X^*_{kl} from a truncated lognormal distribution .
4. Compute $Y^*_\omega = \sum_{k=1}^{K} \sum_{l=1}^{Z^*_{k\omega}} X^*_{kl}$.
5. Repeat the above steps B times to obtain $Y^{*b}_\omega, b = 1, 2, \ldots, B$.
6. The estimate of availability metric, $\hat{u}_p(\omega)$, is computed as the empirical p quantile of Y_ω by $Y_\omega^{*([pB])}$.

Confidence Interval Procedure

1. Simulate $\hat{\boldsymbol{\theta}}^*$ from $\mathrm{N}(\hat{\boldsymbol{\theta}}, \Sigma_{\hat{\boldsymbol{\theta}}})$ and $\hat{\boldsymbol{\psi}}^*$ from $\mathrm{N}(\hat{\boldsymbol{\psi}}, \Sigma_{\hat{\boldsymbol{\psi}}})$.

2. Compute $\hat{\xi}^*_{k\omega}$ with $\hat{\boldsymbol{\theta}}$ replaced by $\hat{\boldsymbol{\theta}}^*$

3. Compute $\hat{u}^*_p(\omega)$ with $\hat{\xi}_{k\omega}$ replaced by $\hat{\xi}^*_{k\omega}$ and $\hat{\boldsymbol{\psi}}$ replaced by $\hat{\boldsymbol{\psi}}^*$.

4. Repeat the above steps B times to obtain $\hat{u}^{*b}_p(\omega)$, $b = 1, 2, \ldots, B$.

5. Compute the one-side upper CI by using $\hat{u}_p^{*([(1-\alpha)B])}(\omega)$.

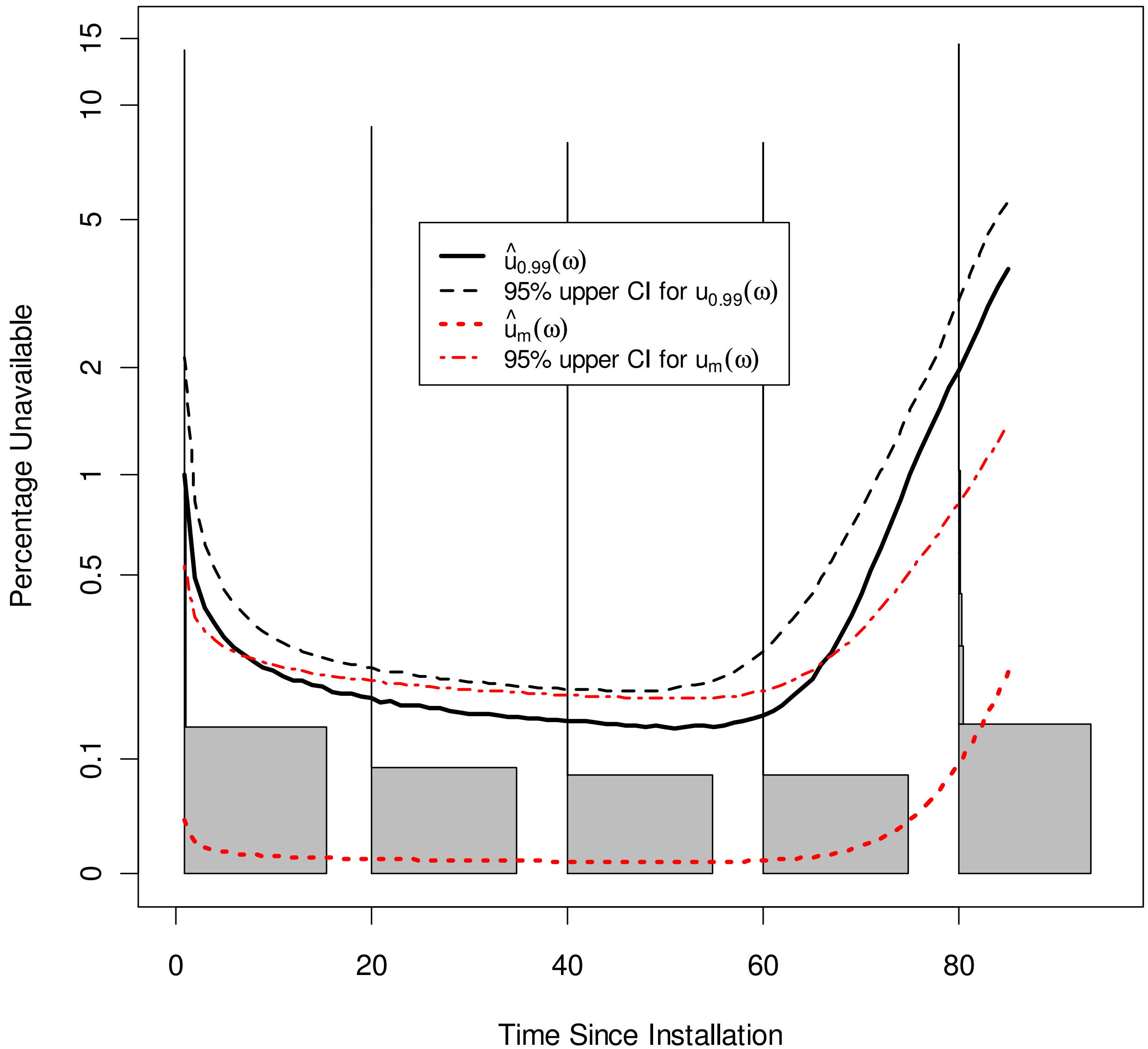
Conditional Analysis

- In some applications, it is also useful to estimate the system unavailability metric for a particular unit that has been in service for a period of time
- This information is useful for maintenance scheduling and system health monitoring for individual units.
- The only difference is to compute $\widehat{\xi}_{k\omega}$ by

$$\widehat{\xi}_{kt_i} = \Lambda_k(t_i + \Delta) - \Lambda_k(t_i).$$

- Here t_i is the current age for unit *i* and $\triangle$ is the future time interval that is of interest.

Percentage of Unavailability for System A



Cost Analysis

❖ The unavailable time of a system will result in expenditures

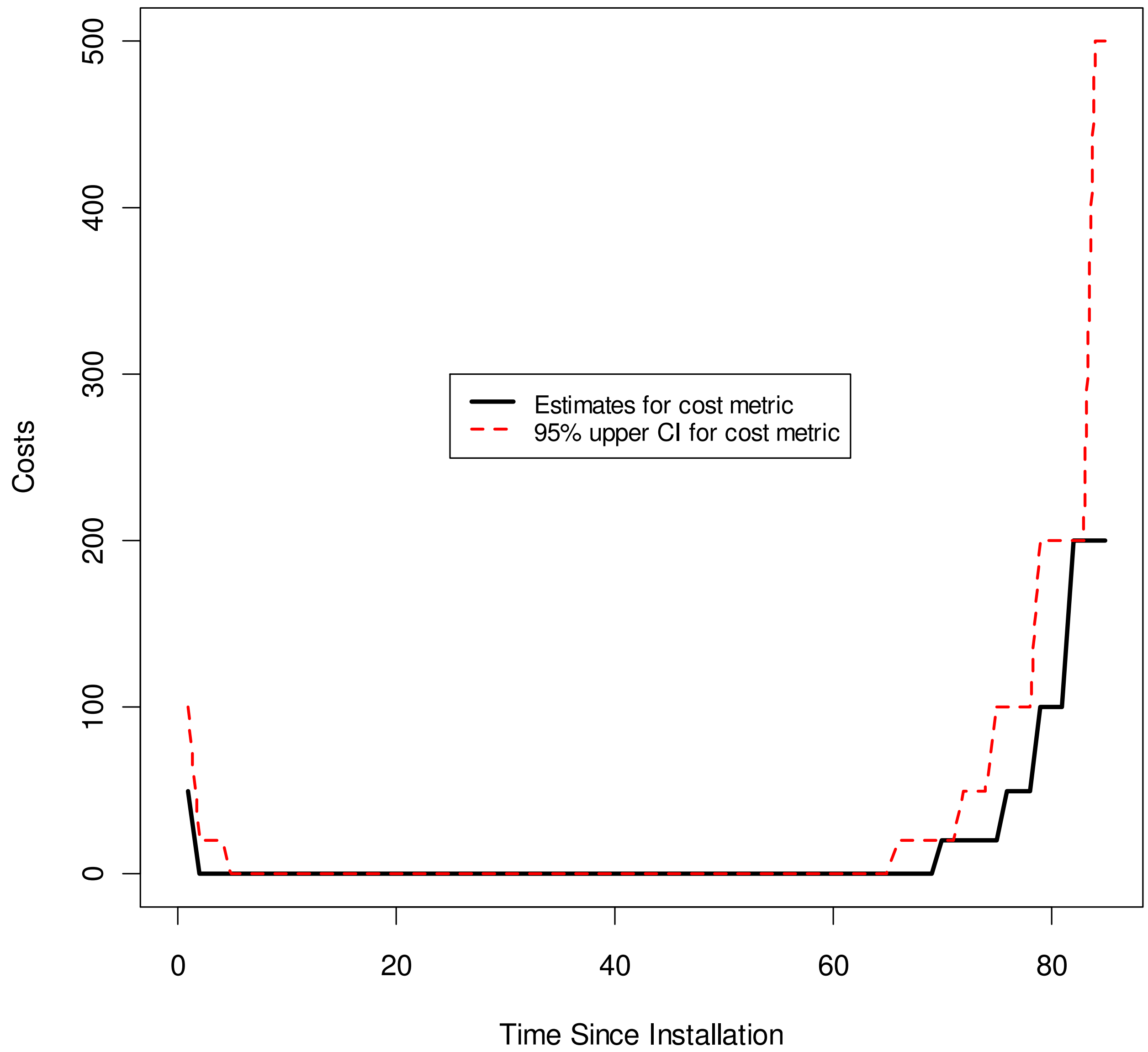
❖ For a given cost function, the cost C_ω can be expressed as $C_\omega = f(Y_\omega)$

❖ A conservative metric for the cost is $f[u_p(\omega)]$, if the cost function is non-decreasing.

❖ Example

$$C_\omega = f(Y_\omega) = \begin{cases} 0, & Y_\omega < 0.5\%; \\ 20, & 0.5\% \le Y_\omega < 1\% \\ 50, & 1\% \le Y_\omega < 1.5\% \\ 100, & 1.5\% \le Y_\omega < 2.5\% \\ 200, & 2.5\% \le Y_\omega < 5\% \\ 500, & Y_\omega \ge 5\% \end{cases} .$$

Cost Analysis for System A



Extension of the Q-Unavailability Metric

- We have discussed the definition of the Q-unavailability metric based on NHPP.
- The definition is generic and the extension to a general repairable system is straightforward.
- Considering a general repairable system, the ith unavailable period is denoted by $[T_i^L, T_i^R], i = 1, 2, \cdots$
- For an interested operation period [L, R], the percentage of the total unavailability time is

$$Y = \frac{\sum_i (\min\{T_i^R, R\} - \max\{T_i^L, L\})}{R - L}$$

- The Q-unavailability metric is defined as $u_p = F_Y^{-1}(p)$

where $F_Y(y) = \Pr(Y \leq y)$ is the cdf.

Conclusions and Areas for future research

- We proposed a new system unavailability metric and developed its estimation based on complicated field data.
- The extended NHPP is used to model the event process.
- The truncated lognormal distribution is used to model the event duration.
- The combination of modeling of event intensity and event duration enables the estimation of unavailability metrics.

- In the future, an alternative approach would be developing nonparametric methods to model the event intensity.
- In the modeling of the time-to-event process, we assumed the System A population is homogeneous.
- It will be interesting, however, to consider population nonhomogeneity in future applications, e.g., frailty model.