Templates for Design Key Construction

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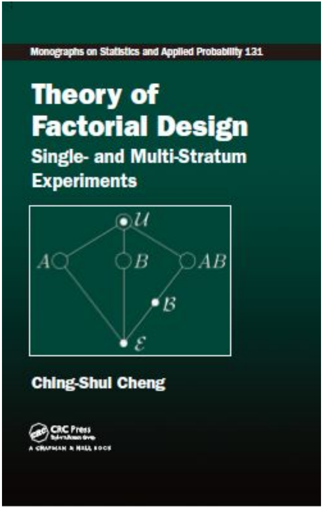
Joint work with Pi-Wen Tsai



Monographs on Statistics and Applied Probability 131
Theory of
Factorial Design
Single- and Multi-Stratum
Experiments
$\mathcal{U}$
A
B
AB
$\mathcal{B}$
$\mathcal{E}$
Ching-Shui Cheng
CRC Press
A CHAPMAN & HALL BOOK

Multistratum Factorial Experiments

- multiple sources of errors, which are determined by the structures of experimental units (blocks, rows, columns, etc.), called block structures or unit structures;
- the treatments have a factorial structure.

Design key

Patterson (1965, 1976), Bailey, (1977), Bailey, Gilchrist and Patterson (1977), Patterson and Bailey (1978)

- A useful unified device in the construction of factorial arrangements and the identification of confounding patterns

Factorial design

Two-level factorial experiments: Each treatment is a combination $(x_1, \cdots, x_n)$ of the levels of n factors, where $x_i = 1, -1$, $i = 1, 2, \cdots, n$.

$y(x_1, x_2, \cdots, x_n) = \alpha(x_1, x_2, \cdots, x_n) + \text{random error}$

$$\alpha(x_1, x_2, \cdots, x_n) = \mu + \sum_{i=1}^{n} \beta_i x_i + \sum_{1 \le i < j \le n} \beta_{ij} x_i x_j$$
$$+ \sum_{1 \le i < j < k \le n} \beta_{ijk} x_i x_j x_k + \cdots + \beta_{12\cdots n} x_1 \cdots x_n$$

β_i: main effect, β_{ij}: two-factor interaction, etc.

Factorial effects

$$\beta_{i_1 \cdots i_k} = \frac{1}{2^n} \left\{ \sum_{(x_1,\ldots,x_n):\ x_{i_1} \cdots\ x_{i_k} = 1} \alpha(x_1, \ldots, x_n) - \sum_{(x_1,\ldots,x_n):\ x_{i_1} \cdots\ x_{i_k} = -1} \alpha(x_1, \ldots, x_n) \right\}$$

Each $\beta_{i_1 \cdots i_k}$ is a treatment contrast.

Each such contrast can be used to divide the treatment combinations into two blocks.

Then it coincides with a block contrast; we say that it is confounded with blocks.

Under a model with fixed block effects (the block effects are unknown constants), the confounded factorial effects cannot be estimated. Under a mixed-effect model with random block effects, the confounded factorial effects are estimated with a larger variance than those that are not confounded with blocks.

Typically we try to avoid confounding main effects with blocks.

To divide the treatment combinations into four blocks, we need two distinct factorial effects.

In general, k (linearly) independent factorial effects can be used to divide the treatment combinations into 2^k blocks.

Suppose the factors are labeled A, B, C, $\ldots$, etc. For two-level factors, we often use a combination of lower case letters to represent a treatment combination. The rule is that a letter is present if and only if the corresponding factor is at level 1.

We also use a combination of capital letters to represent a factorial effect. For example, A represents both factor A and its main effect, and ACD represents the interaction of factors A, C, and D.

A set of factorial effects is independent if none of the effects, when written as a word, can be expressed as a product of some of the other words. For example, AB, ACD, and BCD are dependent since $BCD = (AB)(ACD)$.

An example

Construction of a 2^5 design in eight blocks of size four

Pick three independent interactions, say AC, BD and ABE, to be confounded with block contrasts.

Form the principal (first) block: (1), ace, bde, $abcd$

These are the four treatment combinations that have even numbers of letters in common with each of AC, BD, and ABE.

Once the principal block is determined, the other seven blocks can easily be constructed.

(1)	c	d	cd	e	ce	de	cde
ace	ae	$acde$	ade	ac	a	acd	ad
bde	$bcde$	be	bce	bd	bcd	b	bc
$abcd$	abd	abc	ab	$abcde$	$abde$	$abce$	abe

In addition to AC, BD and ABE, their generalized interactions $(AC)(BD) = ABCD$, $(AC)(ABE) = BCE$, $(BD)(ABE) = ADE$ and $(AC)(BD)(ABE) = CDE$ are also confounded with block contrasts.

unit contrasts and treatment contrasts

treatment contrasts representing factorial effects (main effects and interactions): 31 d.f.

unit contrasts: 31 d.f.

7 d.f.	:	interblock contrasts
24 d.f.	:	intrablock contrasts

two strata

Design key construction is done by, for the main effect of each treatment factor, choosing a unit contrast to be its alias.

Represent the 32 units as combinations of five 2-level unit factors $\mathcal{U}_1$, $\mathcal{U}_2$, $\mathcal{B}_1$, $\mathcal{B}_2$, $\mathcal{B}_3$

interblock contrasts: the effects that only involve $\mathcal{B}_1$, $\mathcal{B}_2$, $\mathcal{B}_3$

intrablock contrasts: the others

We describe the relation between experimental units and the unit factors by a 5×32 matrix $\mathbf{Y}$ such that for each j, $1 \leq j \leq 32$, the jth column of $\mathbf{Y}$ is the level combination of the unit factors corresponding to the jth unit.

We need a 5×32 matrix $\mathbf{X}$ whose jth column gives the treatment combination assigned to the jth unit. The matrix $\mathbf{X}$ then produces the design.

We denote the two levels by 0 and 1, and obtain $\mathbf{X}$ from $\mathbf{Y}$ via

$$\mathbf{X} = \mathbf{KY} \pmod 2$$

where $\mathbf{K}$ is a 5×5 matrix.

The matrix $\mathbf{K}$ connects the treatment factors to the unit factors and is called a design key matrix.

Design key

$$
\begin{array}{cc}
 & \begin{array}{ccccc} \mathcal{U}_1 & \mathcal{U}_2 & \mathcal{B}_1 & \mathcal{B}_2 & \mathcal{B}_3 \end{array} \\
K = & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array}
\end{array}
$$

Two requirements:

1. At least one of the first two entries of each row is not zero (if no treatment main effect is to be confounded with blocks).

2. The columns are linearly independent (under mod 2 arithmetic).

The columns can be used to determine five independent generators of the treatment combinations: ace, bde, c, d, e

(1)	c	d	cd	e	ce	de	cde
ace	ae	$acde$	ade	ac	a	acd	ad
bde	$bcde$	be	bce	bd	bcd	b	bc
$abcd$	abd	abc	ab	$abcde$	$abde$	$abce$	abe

inverse key

$$K^{-1} = \begin{array}{cc} \begin{array}{ccccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} & \mathsf{E} \end{array} & \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix} & \begin{array}{l} \mathcal{U}_1 \\ \mathcal{U}_2 \\ \mathcal{B}_1 \\ \mathcal{B}_2 \\ \mathcal{B}_3 \end{array} \end{array}$$

$K^{-1} = K$

The last three rows of K can be used to determine a set of independent effects that are confounded with blocks: AC, BD, ABE

2^n design in 2^q blocks of size 2^{n-q}

Represent the 2^n units by the combinations of n two-level factors $\mathcal{U}_1, \cdots, \mathcal{U}_{n-q}$, $\mathcal{B}_1, \cdots, \mathcal{B}_q$

Theorem Up to treatment factor relabeling, a 2^n design in 2^q blocks of size 2^{n-q} can be constructed by using the design key

$$\boldsymbol{K} = \begin{bmatrix} \boldsymbol{I}_{n-q} & \boldsymbol{0} \\ \boldsymbol{B} & \boldsymbol{I}_q \end{bmatrix}.$$

where the rows correspond to the treatment factors, the first $n-q$ columns correspond to $\mathcal{U}_1, \cdots, \mathcal{U}_{n-q}$, and the last q columns correspond to $\mathcal{B}_1, \cdots, \mathcal{B}_q$.

Clearly all the columns of $\boldsymbol{K}$ are linearly independent.

No treatment main effect is confounded with block contrasts if and only if all the q rows of $\boldsymbol{B}$ are nonzero.

If the treatment combinations are generated by using the generators determined by the columns of the design key matrix and are arrangeed in the Yates order, then the first 2^{n-q} in the generated sequence are in the same block, and each succeeding set of 2^{n-q} treatment combinations is also in the same block.

The q rows of $\boldsymbol{B}$ need to be chosen. This is equivalent to choosing q independent words to be confounded with block contrasts.

$\boldsymbol{K}^{-1} = \boldsymbol{K}$

The treatment factorial effect defined by each of the last q rows of $\boldsymbol{K}$ is confounded with block contrasts.

No need to

1. check independence of blocking words (The last q rows of $\boldsymbol{K}$ are linearly independent);

2. check that no main effect is confounded with block contrasts;

3. solve equations to find the treatment combinations in the principal block.

Everything is built into the design key matrix

The result also holds for s-level designs where s is a prime power.

$$K^{-1} = \begin{bmatrix} I_{n-q} & \mathbf{0} \\ -B & I_q \end{bmatrix}$$

Split-plot designs

In agricultural experiments, sometimes certain treatment factors require larger plots than others. For example, the machines for sowing seeds require larger plots, but each fertilizer can be applied to a small plot. The seeds are randomly assigned to plots of a suitable size, which are called whole-plots. Each whole-plot is then divided into smaller subplots, and different fertilizers are randomly assigned to the subplots within each whole-plot. The main-effect contrasts of the seed factor (called a whole-plot treatment factor) are confounded with between-whole-plot contrasts since the same seed is applied to all the subplots in each whole-plot.

Many industrial experiments also have a split-plot structure. It may be difficult to change the levels of certain treatment factors, which have to be kept the same on all the experimental runs throughout the day, whereas the levels of the other factors can be varied from run to run. Then each run is a subplot and the experimental runs on the same day constitute a whole-plot.

Same block structure as a block design with each whole-plot as a block and each subplot as a unit in a block.

The main effects of some treatment factors must be confounded with whole-plots (blocks).

Block structure: (4 whole-plots)/(8 subplots)

Five two-level treatment factors

A, B: whole-plot treatment factors

S, T, U: subplot treatment factors.

(1)	(a)	b	ab
stu	$astu$	$bstu$	$abstu$
st	ast	bst	$abst$
u	au	bu	abu
su	asu	bsu	$absu$
t	at	bt	abt
tu	atu	btu	$abtu$
s	as	bs	abs

Block structure: (16 whole-plots)/(2 subplots)

More whole-plots than the # of whole-plot treatment combinations;

Bingham, Schoen, and Sitter (2004)

First construct a design with 4 whole-plots each containing 8 subplots. Then use two interactions involving subplot treatment factors, say ST and SU, to divide each whole-plot into four quarters, each of which is considered as a whole-plot. Then we have 16 whole-plots of size 2.

ST and SU are called independent splitting words (or splitting effects). When choosing the splitting words, one needs to check

1. independence;
2. no subplot treatment main effect gets confounded with whole-plots.

s^n split-plot factorial designs with s^q whole-plots each containing s^{n-q} subplots

s is a prime number or power of a prime number

n_1 of the n treatment factors are whole-plot factors and the other $n_2 = n - n_1$ treatment factors are subplot factors.

The s^n units can be represented by the combinations of n s-level unit factors $\mathcal{W}_1, \ldots, \mathcal{W}_q, \mathcal{S}_1, \ldots, \mathcal{S}_{n-q}$.

Theorem Each complete s^n factorial split-plot design can be constructed by using a design key of the form

$$\boldsymbol{K} = \begin{bmatrix} \boldsymbol{I}_{n-q} & \boldsymbol{0}_{n-q,q} \\ \boldsymbol{B} & \boldsymbol{I}_q \end{bmatrix},$$

where $\boldsymbol{B}$ is $q \times (n-q)$, the first $n-q$ columns of $\boldsymbol{K}$ correspond to $\mathcal{S}_1, \cdots, \mathcal{S}_{n-q}$, and the last q columns correspond to $\mathcal{W}_1, \cdots, \mathcal{W}_q$.

The first $n-q$ rows of $\boldsymbol{K}$ correspond to subplot treatment factors.
The next n_1 rows correspond to whole-plot treatment factors, and if $n_1 < q$, the last $q - n_1$ rows correspond again to sub-plot treatment factors.

All the first n_1 rows of $\boldsymbol{B}$ are zero, and if $n_1 < q$, the last $q - n_1$ rows of $\boldsymbol{B}$ are nonzero.

Represent the 32 units as combinations of five two-level unit factors $\mathcal{W}_1, \mathcal{W}_2, \mathcal{W}_3, \mathcal{W}_4, \mathcal{S}$.

$$
\begin{array}{cc}
 & \begin{array}{ccccc} \mathcal{S} & \mathcal{W}_1 & \mathcal{W}_2 & \mathcal{W}_3 & \mathcal{W}_4 \end{array} \\
\boldsymbol{K} = & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ * & 1 & 0 & 0 & 0 \\ * & 0 & 1 & 0 & 0 \\ * & 0 & 0 & 1 & 0 \\ * & 0 & 0 & 0 & 1 \end{bmatrix}
\begin{array}{c} S \\ A \\ B \\ T \\ U \end{array}
\end{array}
$$

$$\begin{array}{cc} & \mathcal{S}\ \mathcal{W}_1\mathcal{W}_2\mathcal{W}_3\mathcal{W}_4 \\ K = & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{c} S \\ A \\ B \\ T \\ U \end{array} \end{array}$$

There is only one design up to isomorphism.

Five independent generators stu, a, b, t, and u are identified from the columns of $\boldsymbol{K}$.

(1)	a	b	ab	t	at
stu	$astu$	$bstu$	$abstu$	su	asu

etc.

Blocked split-plot designs

s^n design with s^q whole-plots each containing s^{n-q} subplots, where there are n_1 whole-plot treatment factors, $n_2 = n - n_1$ subplot treatment factors, and the s^q whole-plots are divided into s^g blocks each of size s^{q-g}.

Block structure: $s^g/s^{q-g}/s^{n-q}$

McLeod and Brewster (2004) proposed three construction methods: pure whole-plot blocking, separation, and mixed blocking.

Represent the s^n units by the combinations of n s-level unit factors

$\mathcal{S}_1, \cdots, \mathcal{S}_{n-q}, \mathcal{W}_1, \cdots, \mathcal{W}_{q-g}, \mathcal{B}_1, \cdots, \mathcal{B}_g$

Three strata

A treatment main effect is confounded with a block (respectively, whole-plot) contrast if its unit alias does not involve any of $\mathcal{S}_1, \cdots, \mathcal{S}_{n-q}$, $\mathcal{W}_1, \cdots, \mathcal{W}_{q-g}$ (respectively, involves at least one of $\mathcal{W}_1, \cdots, \mathcal{W}_{q-g}$ but not any of $\mathcal{S}_1, \cdots, \mathcal{S}_{n-q}$), and is orthogonal to block and whole-plot contrasts if its unit alias involves at least one of $\mathcal{S}_1, \cdots, \mathcal{S}_{n-q}$.

$$\boldsymbol{K} = \begin{bmatrix} \boldsymbol{I}_{n-q} & \boldsymbol{0}_{n-q,q-g} & \boldsymbol{0}_{n-q,g} \\ & \boldsymbol{I}_{q-g} & \boldsymbol{0}_{q-g,g} \\ \boldsymbol{B} & & \\ & \boldsymbol{C} & \boldsymbol{I}_g \end{bmatrix},$$

where $\boldsymbol{B}$ is $q \times (n-q)$, $\boldsymbol{C}$ is $g \times (q-g)$, the first $n-q$ columns of $\boldsymbol{K}$ correspond to $\mathcal{S}_1, \cdots, \mathcal{S}_{n-q}$, the next $q-g$ columns correspond to $\mathcal{W}_1, \cdots, \mathcal{W}_{q-g}$, the last g columns correspond to $\mathcal{B}_1, \cdots, \mathcal{B}_g$, the first $n-q$ rows of $\boldsymbol{K}$ correspond to subplot treatment factors, the next n_1 rows correspond to whole-plot treatment factors, and all the remaining rows correspond to subplot treatment factors if $n_1 < q$.

Furthermore, the first n_1 rows of $\boldsymbol{B}$ are zero, all the last $q-n_1$ rows of $\boldsymbol{B}$ are nonzero if $n_1 < q$, and all the first $n_1-(q-g)$ rows of $\boldsymbol{C}$ are nonzero if $n_1 > q-g$.

For constructing an s^{n-p} fractional factorial design, we can add p rows to a design key for the complete factorial of $n-p$ basic factors, one for each added factor.

Two treatment factorial effects are aliased if and only if their unit aliases are aliased.

Given the defining relation of the fraction, the row of the design key associated with each added factor can be obtained as a linear combination of the rows corresponding to the basic factors whose interaction is used to define the given factor.

Need to check that the constraints imposed by the block structures are observed. For instance, if no treatment main effects are to be confounded with block contrasts, then one needs to make sure their unit aliases are not block contrasts.

Experiments with multiple processing stages

Levels of treatment factors are assigned at different stages. At each stage, the experimental units are divided into disjoint classes. All the units in the same class are assigned the same level of each factor whose levels are set at that stage.

For example, the fabrication of integrated circuits involves a sequence of processing stages. At each stage several wafers may be processed together and they are all assigned the same levels of some treatment factors.

2^4 experiment, four stages, four classes at each stage

0000	0011	0101	0110
0010	0001	0111	0100
1001	1010	1100	1111
1011	1000	1110	1101

1	2	3	4
2	1	4	3
3	4	1	2
4	3	2	1

1	2	3	4
4	3	2	1
2	1	4	3
3	4	1	2

2^4 experiment, four stages, four calsses at each stage

0000	0011	0101	0110
0010	0001	0111	0100
1001	1010	1100	1111
1011	1000	1110	1101

1	2	3	4
2	1	4	3
3	4	1	2
4	3	2	1

1	2	3	4
4	3	2	1
2	1	4	3
3	4	1	2

2^4 experiment, four stages, four classes at each stage

0000	0011	0101	0110
0010	0001	0111	0100
1001	1010	1100	1111
1011	1000	1110	1101

1	2	3	4
2	1	4	3
3	4	1	2
4	3	2	1

1	2	3	4
4	3	2	1
2	1	4	3
3	4	1	2

2^4 experiment, four stages, four classes at each stage

0000	0011	0101	0110
0010	0001	0111	0100
1001	1010	1100	1111
1011	1000	1110	1101

1	2	3	4
2	1	4	3
3	4	1	2
4	3	2	1

1	2	3	4
4	3	2	1
2	1	4	3
3	4	1	2

2^4 experiment, four stages, four classes at each stage

0	0	0	0	1	1	1	1
1	**0**	**0**	**1**	3	1	3	2
0	**0**	**1**	**0**	2	1	2	4
1	0	1	1	4	1	4	3
0	**1**	**0**	**1**	1	3	3	3
1	1	0	0	3	3	1	4
0	1	1	1	2	3	4	2
1	1	1	0	4	3	2	1
0	**0**	**1**	**1**	1	2	2	2
1	0	1	0	3	2	4	1
0	0	0	1	2	2	1	3
1	0	0	0	4	2	3	4
0	1	1	0	1	4	4	4
1	1	1	1	3	4	2	3
0	1	0	0	2	4	3	1
1	1	0	1	4	4	1	2

Wu (1989) showed that for even n, the $2^n - 1$ columns of a saturated regular 2-level design of size 2^n can be decomposed into $(2^n - 1)/3$ disjoint groups of size three such that any column is the sum of the other two columns in the same group. For odd n, one can only construct up to $(2^n - 5)/3$ disjoint groups.

1	2	3	4
0	0	0	0
1	0	0	0
0	1	0	0
1	1	0	0
0	0	1	0
1	0	1	0
0	1	1	0
1	1	1	0
0	0	0	1
1	0	0	1
0	1	0	1
1	1	0	1
0	0	1	1
1	0	1	1
0	1	1	1
1	1	1	1

1	2	12	3	4	34	13	24	1234	23	124	134	14	123	234
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	0	0	0	1	0	1	0	1	1	1	1	0
0	1	1	0	0	0	0	1	1	1	1	0	0	1	1
1	1	0	0	0	0	1	1	0	1	0	1	1	0	1
0	0	0	1	0	1	1	0	1	1	0	1	0	1	1
1	0	1	1	0	1	0	0	0	1	1	0	1	0	1
0	1	1	1	0	1	1	1	0	0	1	1	0	0	0
1	1	0	1	0	1	0	1	1	0	0	0	1	1	0
0	0	0	0	1	1	0	1	1	0	1	1	1	0	1
1	0	1	0	1	1	1	1	0	0	0	0	0	1	1
0	1	1	0	1	1	0	0	0	1	0	1	1	1	0
1	1	0	0	1	1	1	0	1	1	1	0	0	0	0
0	0	0	1	1	0	1	1	0	1	1	0	1	1	0
1	0	1	1	1	0	0	1	1	1	0	1	0	0	0
0	1	1	1	1	0	1	0	1	0	0	0	1	0	1
1	1	0	1	1	0	0	0	0	0	1	1	0	1	1

$$(0,0,0) \to 1, (0,1,1) \to 2, (1,0,1) \to 3, (1,1,0) \to 4$$

1	2	3	4	1, 2, 12	3, 4, 34	13, 24, 1234	23, 124, 134
0	0	0	0	1	1	1	1
1	0	0	0	3	1	3	2
0	1	0	0	2	1	2	4
1	1	0	0	4	1	4	3
0	0	1	0	1	3	3	3
1	0	1	0	3	3	1	4
0	1	1	0	2	3	4	2
1	1	1	0	4	3	2	1
0	0	0	1	1	2	2	2
1	0	0	1	3	2	4	1
0	1	0	1	2	2	1	3
1	1	0	1	4	2	3	4
0	0	1	1	1	4	4	4
1	0	1	1	3	4	2	3
0	1	1	1	2	4	3	1
1	1	1	1	4	4	1	2

Choose one contrast from each groupto be relabeled as 1, 2, 3, and 4, respectively.

Suppose 1, 3, 24, and 134 are to be relabeled as 1, 2, 3, and 4, respectively.

$$\mathsf{K} = \begin{array}{cc} & \begin{array}{cccc} \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} \end{array} \\ & \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{array}\right] \begin{array}{c} \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \\ \mathbf{4} \end{array} \end{array}.$$

1	2	3	4	1, 2, 12	3, 4, 34	13, 24, 1234	23, 124, 134
0	0	0	0	1	1	1	1
1	0	0	0	3	1	3	2
0	1	0	0	2	1	2	4
1	1	0	0	4	1	4	3
0	0	1	0	1	3	3	3
1	0	1	0	3	3	1	4
0	1	1	0	2	3	4	2
1	1	1	0	4	3	2	1
0	0	0	1	1	2	2	2
1	0	0	1	3	2	4	1
0	1	0	1	2	2	1	3
1	1	0	1	4	2	3	4
0	0	1	1	1	4	4	4
1	0	1	1	3	4	2	3
0	1	1	1	2	4	3	1
1	1	1	1	4	4	1	2

Replace the generators with those obtained from the columns of K.

1	2	3	4	1, 1234, 234	2, 124, 14	12, 3, 123	134, 13, 4
0	0	0	0	1	1	1	1
1	0	0	1	3	1	3	2
0	0	1	0	2	1	2	4
1	0	1	1	4	1	4	3
0	1	0	1	1	3	3	3
1	1	0	0	3	3	1	4
0	1	1	1	2	3	4	2
1	1	1	0	4	3	2	1
0	0	1	1	1	2	2	2
1	0	1	0	3	2	4	1
0	0	0	1	2	2	1	3
1	0	0	0	4	2	3	4
0	1	1	0	1	4	4	4
1	1	1	1	3	4	2	3
0	1	0	0	2	4	3	1
1	1	0	1	4	4	1	2

Cheng and Tsai (2013), Statistica Sinica

Cheng (2013), Theory of Factorial Design