The Augmented Desirability Function: Methods and Applications

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- Response surface methodology

  define -> design -> modeling -> estimation ->

  optimization -> forecasting ->confirmation

  - optimization: determine operating conditions on a set of input factors (x) that result in an optimum response (y)
  - **multiple response optimization**
  - **desirability function**

- Application
  - industry: process and product optimization or quality improvement

Multiple response optimization

- chemical process: several properties of the product output
- consumer products (food, tobacco): taste as a response, undesirable byproducts.
- pharmaceutical or biomedical area: efficacy of the drug or remedy, the possibility of serious side effects
- tool life problem: cutting speed and depth of cut influence the primary response (the life of the tool), a secondary response (rate of metal removed)

Outline

- Overview

I. An augmented approach to the desirability function

II. Balancing location and dispersion effects for multiple responses

- Summary

Overview

Challenges of multiple response optimization

- Different units of measurement
- Different criteria: larger-the-better (LTB)
  smaller-the-better (STB)
  nominal-the-best (NTB)
- Not be of equal importance

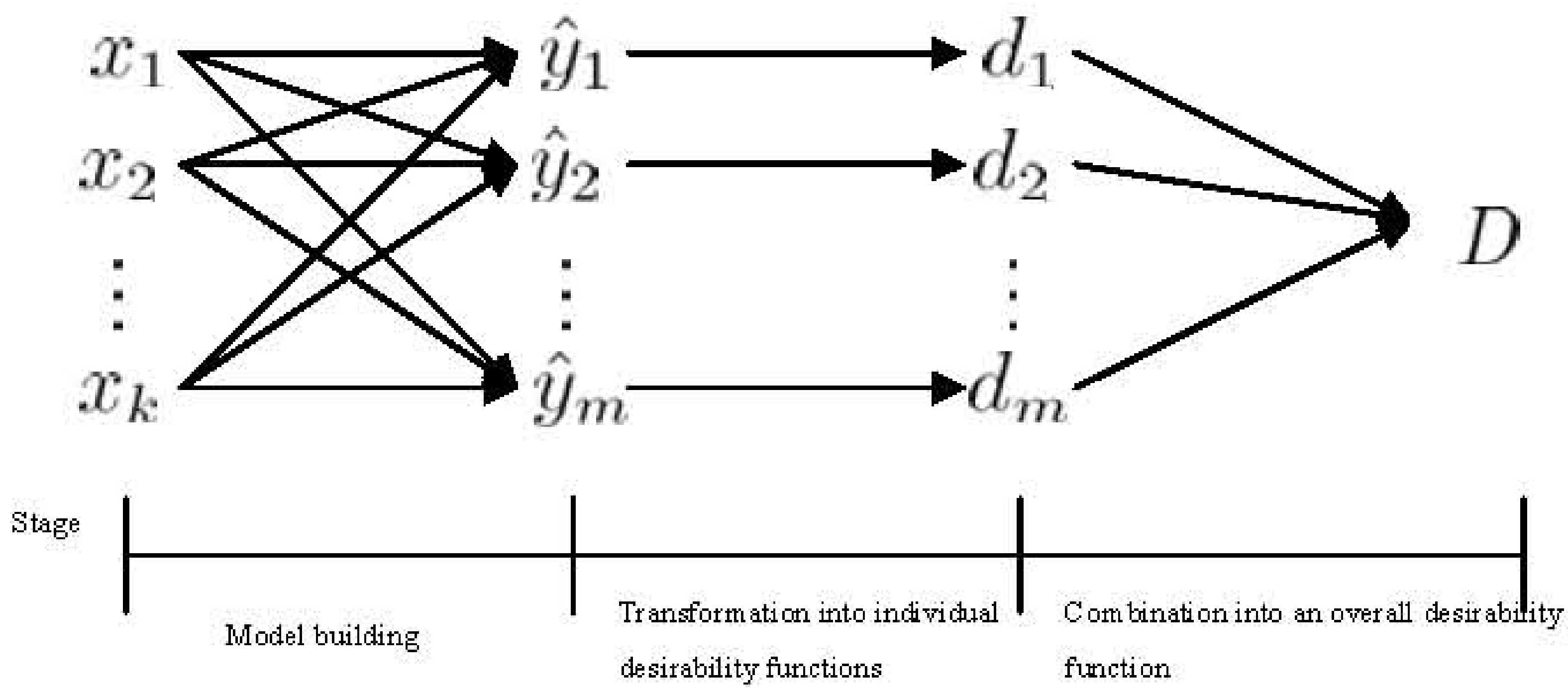
Response		Type
y1	stability (log(seconds))	LTB
y2	volumetric ratio (none(ratio))	STB
y3	temperature (°C)	NTB
Factor		
x1	concentration of surfactant	
x2	concentration of salt	
x3	time of stirring	

Colloidal gas aphrons experiment (Jauregi et al., 1997)

Multiple response techniques

- Desirability function (Harrington 1965)
- Dual response approach (Myers and Carter, 1973): primary and secondary responses
- Generalized distance (Khuri and Conlon, 1981): an estimated distance from the idea design point

The desirability function
(Harrington, 1965)

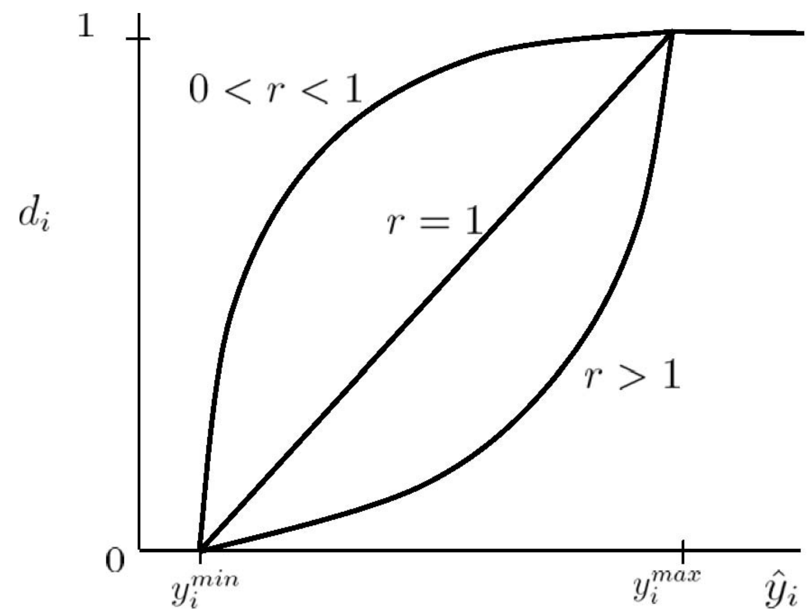


- Individual desirability function $d_i,\ 0 \le d_i \le 1$
- An overall desirability function $D,\ 0 \le D \le 1$
- Goal: find settings of x to maximize D

Transformation

Derringer and Suich, 1980
(a) The larger-the-better type

$$d_i = \begin{cases} 0 & \text{for } \hat{y}_i \le y_i^{min} \\ \left(\frac{\hat{y}_i - y_i^{min}}{y_i^{max} - y_i^{min}} \right)^r & \text{for } y_i^{min} < \hat{y}_i < y_i^{max} \\ 1 & \text{for } \hat{y}_i \ge y_i^{max} \end{cases}$$

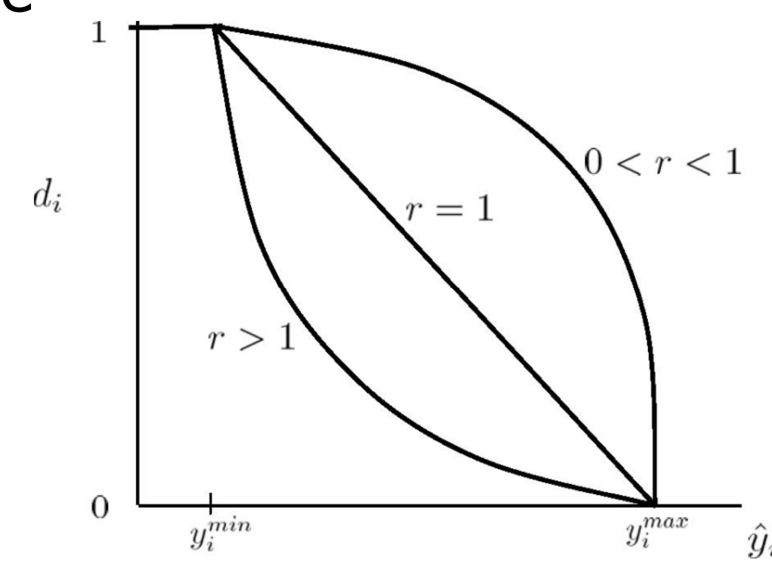


- bounds: $[y_i^{min}, y_i^{max}]$
- shape parameter: r
- linear transformation: $r = 1$
- non-linear transformation: $r > 1$

  $0 < r < 1$

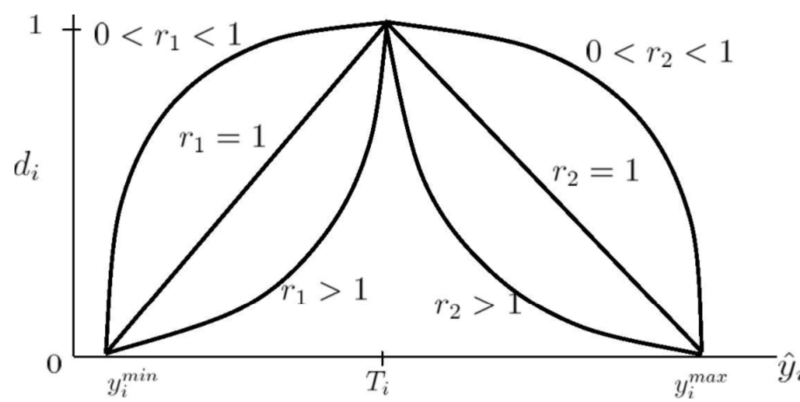
(b) The smaller-the-better type

$$d_i = \begin{cases} 1 & \text{for } \hat{y}_i \le y_i^{min} \\ \left(\frac{y_i^{max} - \hat{y}_i}{y_i^{max} - y_i^{min}}\right)^r & \text{for } y_i^{min} < \hat{y}_i < y_i^{max} \\ 0 & \text{for } \hat{y}_i \ge y_i^{max} \end{cases}$$



(c) The nominal-the-best type (T_i is the target)

$$d_i = \begin{cases} \left(\frac{\hat{y}_i - y_i^{min}}{T_i - y_i^{min}}\right)^{r_1} & \text{for } y_i^{min} \le \hat{y}_i \le T_i \\ \left(\frac{y_i^{max} - \hat{y}_i}{y_i^{max} - T_i}\right)^{r_2} & \text{for } T_i < \hat{y}_i \le y_i^{max} \\ 0 & \text{for } \hat{y}_i < y_i^{min} \ or\ \hat{y}_i > y_i^{max} \end{cases}$$



Combination

- The geometric mean (Harrington, 1965)

$$D = (d_1 d_2 \cdots d_m)^{1/m}$$

- Weighted geometric mean (Derringer, 1994)

$$D_w = d_1^{w_1} d_2^{w_2} \cdots d_m^{w_m}$$

$$0 < w_i < 1, \ \sum w_i = 1$$

Interpretation of desirability values

Value	Interpretation
1	Better outcome has no preference
1-0.8	Acceptable and excellent
0.8-0.63	Acceptable and good
0.63-0.4	Acceptable but poor
0.4-0.3	Borderline
0.3-0	Unacceptable
0	Completely unacceptable

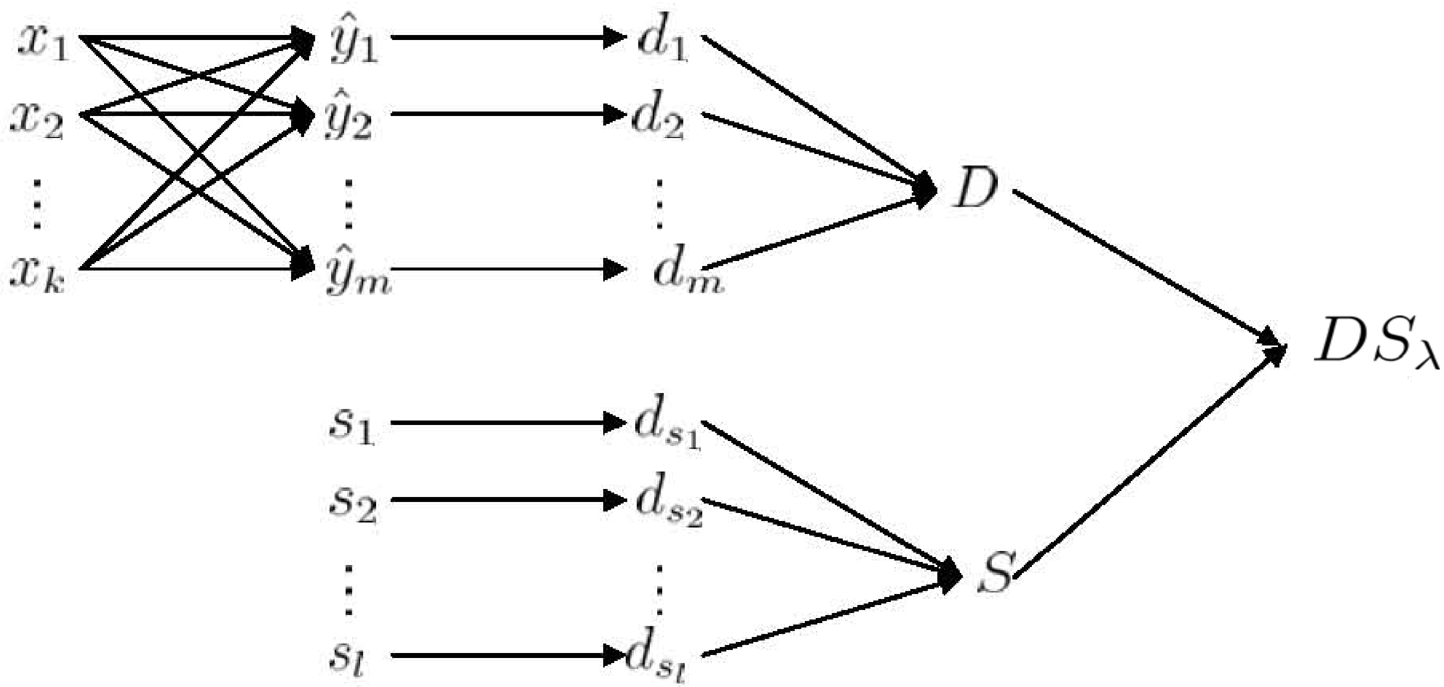
A higher value of desirability represents greater improvement

Harrington (1965)

The augmented desirability function

- Motivation
  - the need for a new desirability function approach that accounts for some secondary information to make the optimal solution more practical
- Underlying idea
  - incorporate desirability functions of some secondary information into the overall desirability function via combination

The augmented desirability function



- The secondary information s
- Secondary individual desirability $d_s, \ 0 \le d_s \le 1$
- The secondary overall desirability $S, \ 0 \le S \le 1$
- Goal: find settings of x to max $DS_\lambda, \ 0 \le DS_\lambda \le 1$

The augmented desirability function

$$DS_\lambda = D^\lambda S^{1-\lambda} = (d_1 d_2 \cdots d_m)^{\lambda/m} (d_{s_1} d_{s_2} \cdots d_{s_l})^{(1-\lambda)/l}$$

- A weighted geometric mean of D and S
- Weight $\lambda,\ 0 < \lambda \leq 1$
- Harrington's desirability function: $\lambda = 1$
- Goal: for each λ, to find an optimal solution x that maximizes DS_λ over the experimental region

Find the optimal solution on x

- adapt the `desirability` package in R to incorporate the secondary desirability functions

- The `optim` function contains the Nelder-Mead direct search algorithm for maximizing the desirability

Confirmation

- When the optimum condition on x had been determined, the researcher should do confirmatory runs at that condition to be sure that all responses are in a satisfactory region.

The secondary information

- Section I
  - prediction variance
- Section II
  - dispersion effect

I. An augmented approach to the desirability function

Hsiu-Wen Chen, Weng Kee Wong and Hongquan Xu (2012)

Journal of Applied Statistics, 39, 599-613.

For minimizing the prediction variance

Outline of Section I

- Why do we minimize the prediction variance?
- How to add the prediction variance to the desirability function?
- Data application

Why do we minimize the prediction variance?

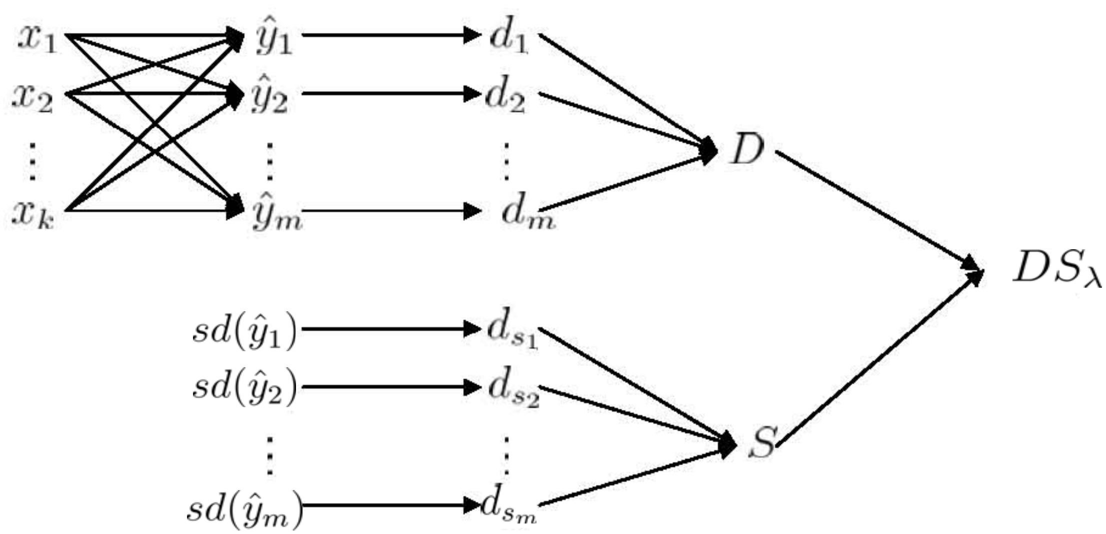
A narrower prediction interval

- The desirability function ignores the variability in each predicted response.
- The actual response at the optimal solution sometimes may fall outside the acceptable region.
- If the desired range of the response does not cover the prediction interval, there is a large chance that the actual response at the optimal solution will not be acceptable.

Optimum solution	Response	$\hat{y}_i$	90% Prediction interval	Bound
$(x_1, x_2) = (19.16, 50)$	y_1	1.77	(1.17, 2.37)	[1.58, 3.32]
	y_2	1.06	(0.77, 1.36)	[1, 1.5]
	y_3	9.39	(7.87, 10.91)	[6.44, 12.09]
	y_4	15.22	(12.18, 18.27)	[10.64, 20.05]
	y_5	73.85	(65.9, 81.85)	[66, 100]

How to add the prediction variance to the desirability function?

The augmented desirability function



$$DS_\lambda = D^\lambda S^{1-\lambda} = (d_1 d_2 \cdots d_m)^{\lambda/m} (d_{s_1} d_{s_2} \cdots d_{s_m})^{(1-\lambda)/m}$$

- minimizing the variances of the predicted responses

Prediction variance

- Factors $\boldsymbol{x} = (x_1, x_2, \ldots, x_k)$ affect each of the responses
- Assume the ith response may be approximated by a linear regression model:

  $$y_i = f_i(\boldsymbol{x})^T \boldsymbol{\beta}_i + \varepsilon_i \qquad i = 1, 2, \ldots, m,$$

  where $\boldsymbol{\beta}_i$ is the vector of p_i unknown regression parameters, $f_i(\boldsymbol{x})$ is the $p_i \times 1$ vector of the coefficients, ε_i is a normal random error, and $\varepsilon_i \sim N(0, \sigma_i^2)$.
- Each predicted response at the point $\boldsymbol{x}$ is $\hat{y}_i = f_i(\boldsymbol{x})^T \hat{\boldsymbol{\beta}}_i$ and has variance $var(\hat{y}_i) = \sigma_i^2 v_i(\boldsymbol{x}),$

  where $v_i(\boldsymbol{x}) = f_i(\boldsymbol{x})^T (X_i^T X_i)^{-1} f_i(\boldsymbol{x})$

  and X_i is the $n \times p_i$ model matrix

The secondary individual desirability function

- Transform each standard deviation of the predicted response

  $$sd(\hat{y}_i) = \sqrt{var(\hat{y}_i)} = \sigma_i \sqrt{v_i(\boldsymbol{x})}$$

  into the secondary individual desirability function d_{s_i}
- $sd(\hat{y}_i)$ is considered as the smaller-the-better type
- The lower bound: 0
- The upper bound: $c_i \sigma_i$, where c_i is a user-selected constant
- a smaller value of c_i implies a narrower range for the values of $sd(\hat{y}_i)$

The secondary individual desirability function

$$d_{s_i} = \begin{cases} \left(\frac{c_i\sigma_i - sd(\hat{y}_i)}{c_i\sigma_i}\right)^r = \left(\frac{c_i - \sqrt{v_i(\boldsymbol{x})}}{c_i}\right)^r & \text{for } 0 < v_i(\boldsymbol{x}) < c_i^2 \\ 0 & \text{for } v_i(\boldsymbol{x}) \geq c_i^2, \end{cases}$$

where r is the shape parameter ($r > 0$)

$v_i(\boldsymbol{x}) = f_i(\boldsymbol{x})^T (X_i^T X_i)^{-1} f_i(\boldsymbol{x})$

- $c_i = \sqrt{v_i(\boldsymbol{x}_*)}$ where $\boldsymbol{x}_*$ is the optimal solution from Harrington's desirability function
- interpretation: d_{s_i} is the relative change in standard deviation between the new optimal solution and the one from Harrington's desirability function approach when $r = 1$

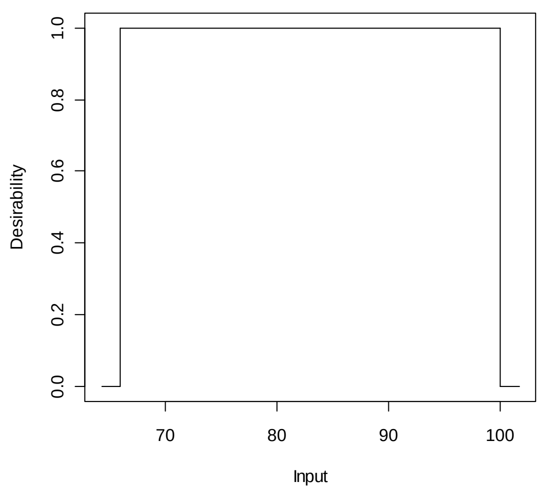
Data application

Pharmaceutical preparations problem

- Candioti et al. (2006) analyzed this data set using Harrington's desirability function
- Goal: find factor settings that optimize five responses

Response		Type	Bounds
y1	resolution 1 for active ingredients	STB	(1.58, 3.32)
y2	resolution 2 for active ingredients	NTB	(1, 1.5)
y3	resolution 3 for active ingredients	STB	(6.44, 12.09)
y4	analysis time	STB	(10.64, 20.05)
y5	capillary current	Range	(66, 100)
Factor			
x1	voltage		
x2	buffer solution concentration		

target for y2: 1.125



- central composite design with five center points in a randomized order

- experimental region:

$$x_1 = [14.9, 20]$$

$$x_2 = [50, 101]$$

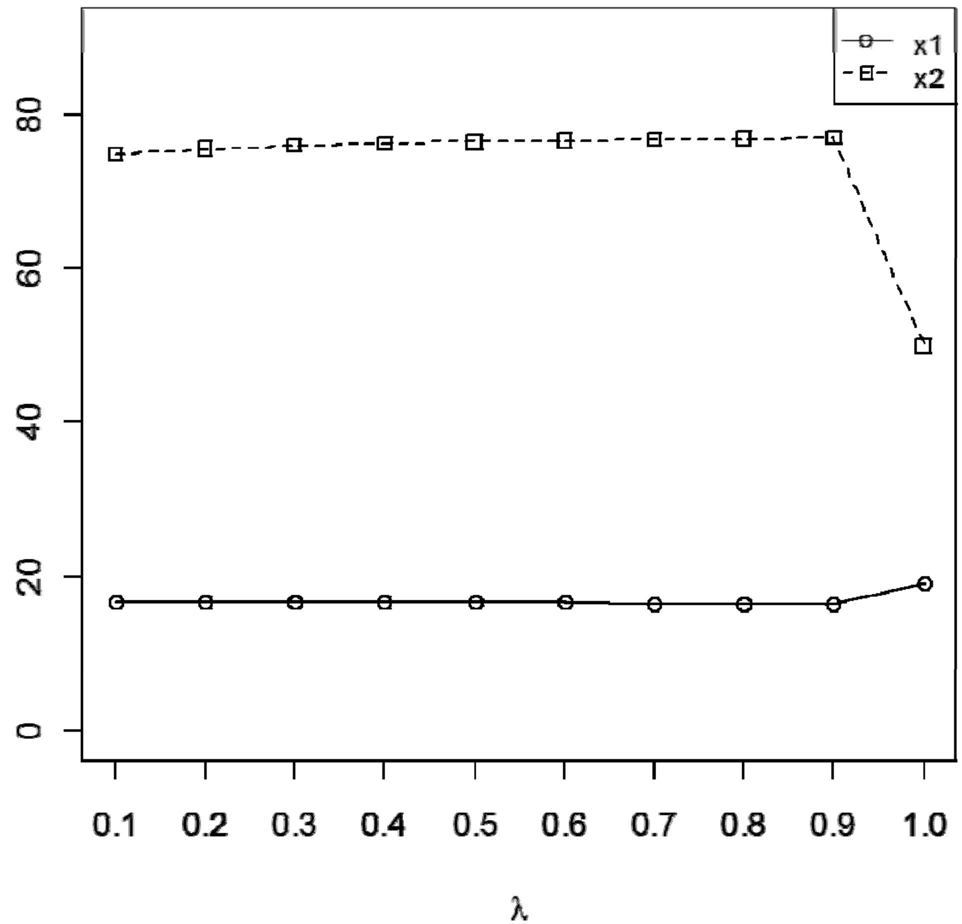
x_1	x_2	y_1	y_2	y_3	y_4	y_5
17.5	50.34	2.86	1.23	10.03	19.54	66
15.7	56.70	2.40	1.10	10.28	18.19	66
19.3	56.70	1.78	0.88	8.35	12.57	90
14.9	75.52	3.18	1.36	12.09	20.05	80
17.5	75.52	2.32	0.95	8.00	14.97	110
17.5	75.52	2.40	0.92	7.22	14.44	110
17.5	75.52	2.11	0.88	7.34	14.59	110
17.5	75.52	2.38	1.08	8.47	14.40	110
17.5	75.52	2.56	1.15	8.54	14.47	110
20.0	75.52	1.58	0.79	6.44	10.64	150
15.7	93.64	3.32	1.50	11.31	17.79	125
19.3	93.64	2.00	0.99	7.17	10.99	205
17.5	100.69	2.72	1.34	9.54	13.47	175

Models & Transformation

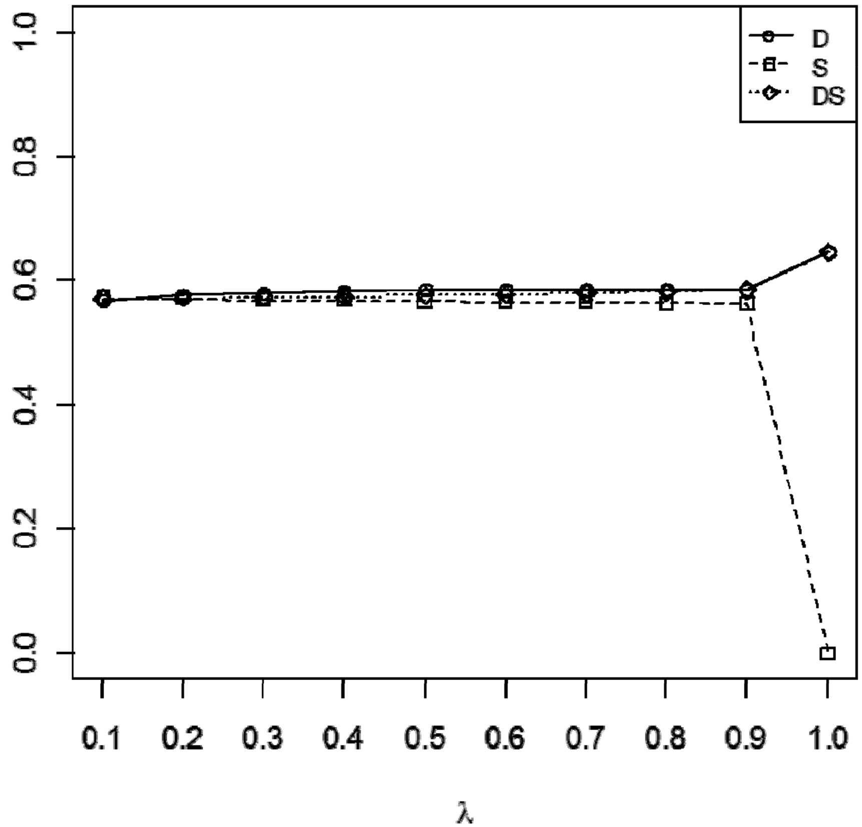
Following Candioti et al., 2006

- Fit y1 using a first order linear model and fit y2, y3, y4 and y5 using quadratic models

- All shape parameters are set at 1

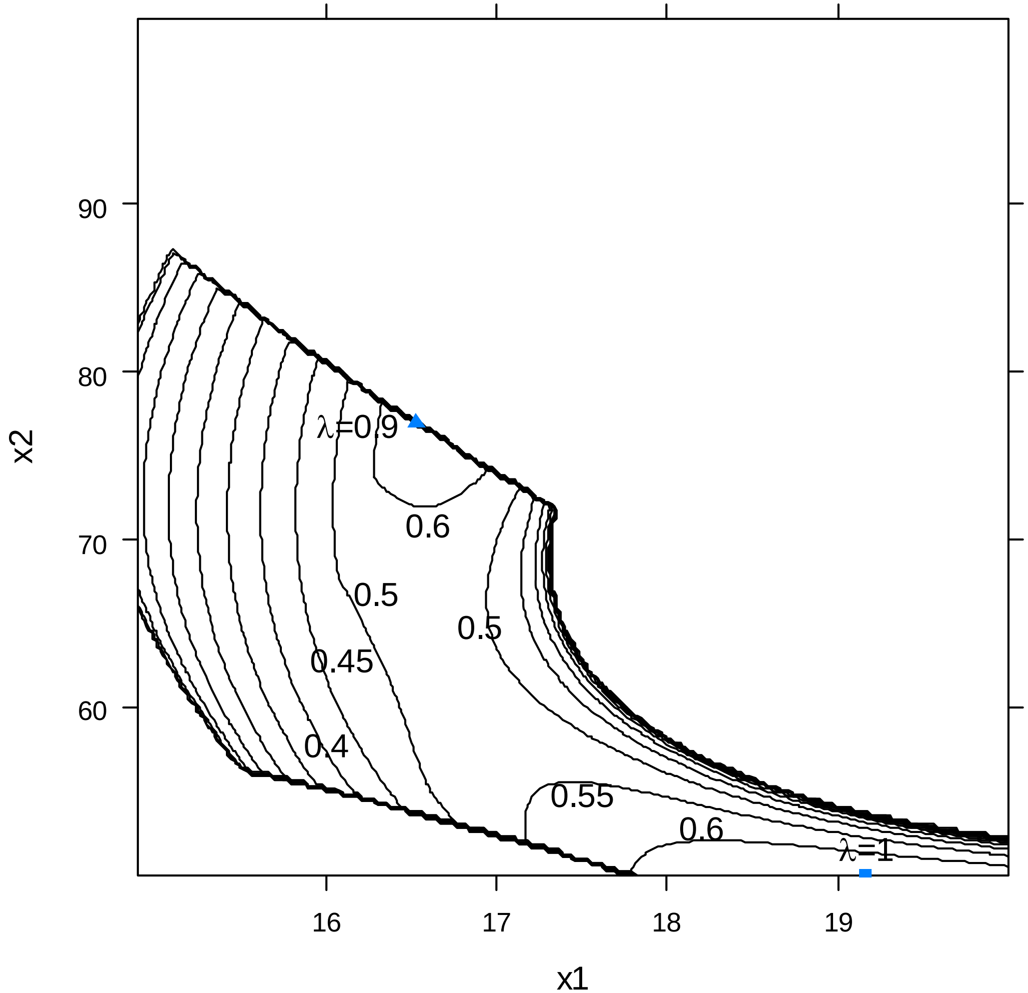


Optimal solutions are quite different when $\lambda \leq 0.9$ and when $\lambda = 1$

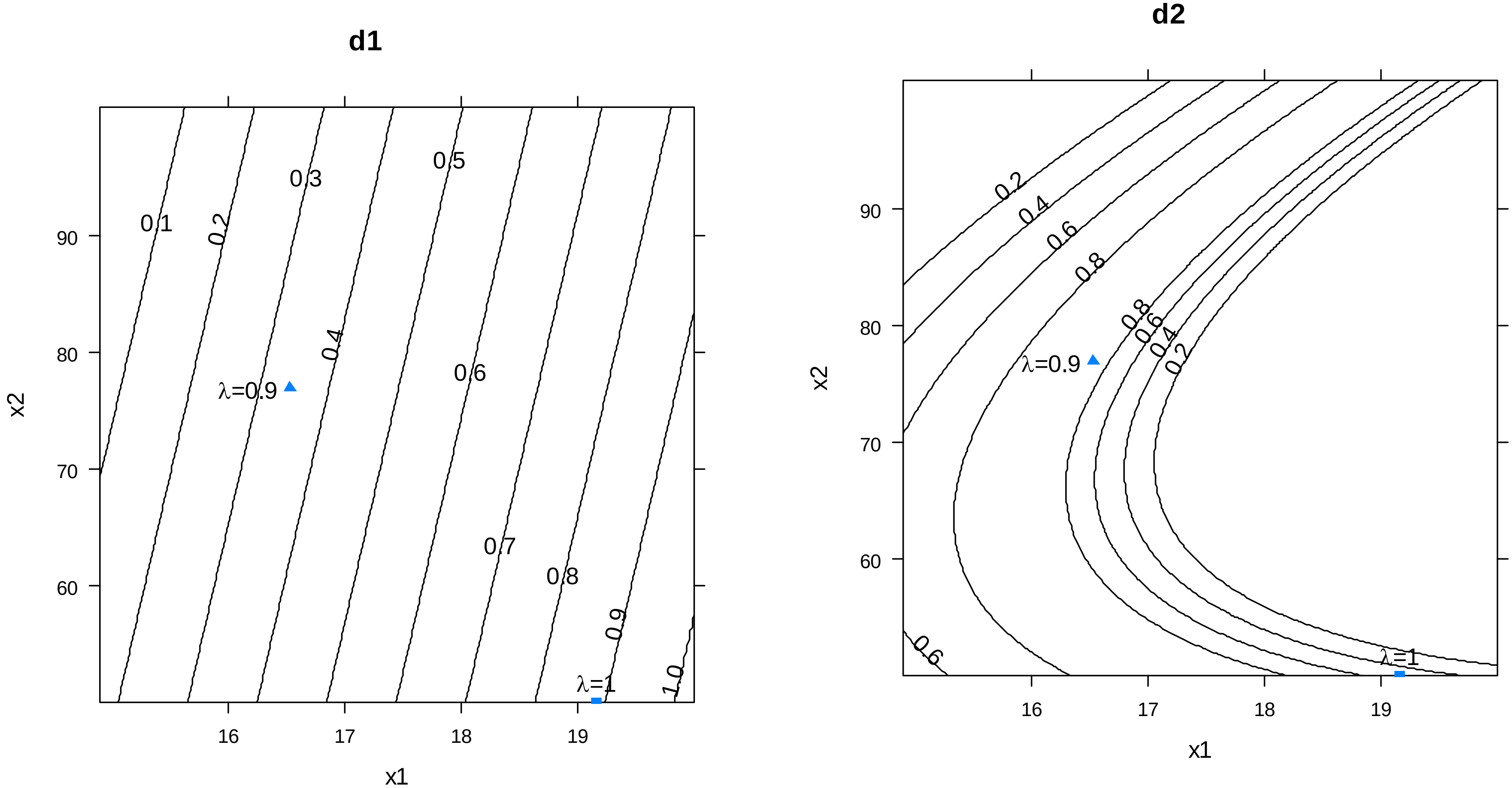


- The values of S are larger when $\lambda \leq 0.9$, which indicates that the augmented approach produces smaller $sd(\hat{y}_i)$

- D, S, and DS are almost the same when $\lambda = 0.1 \sim 0.9$, but they are quite different with those from Harrington's desirability function ($\lambda = 1$)



- The optimal solutions are in different areas of the saddle surface
- $\lambda = 1$ is around the boundary of the experimental region
- $\lambda = 0.9$ is much closer to the center of the experimental region



The difference arises from d1 and d2

Optimization results	Response	$\hat{y}_i$	d_i	$sd(\hat{y}_i)$	d_{s_i}	90% Prediction interval	Length
Harrington's desirability							
$\lambda = 1$, x_* = (19.16, 50)	y_1	1.77	0.89	0.18		(1.17, 2.37)	1.20
$D = 0.6454$	y_2	1.06	0.52	0.11		(0.77, 1.36)	0.59
	y_3	9.39	0.48	0.58		(7.87, 10.91)	3.04
	y_4	15.2	0.51	1.17		(12.18, 18.27)	6.09
	y_5	73.9	1.00	3.05		(65.90, 81.85)	15.95
Augmented approach							
$\lambda = 0.9$, x_* = (16.53, 76.97)	y_1	2.72	0.34	0.09	0.49	(2.19, 3.25)	1.06
$D = 0.5874$	y_2	1.12	0.94	0.05	0.58	(0.90, 1.34)	0.44
$S = 0.5635$	y_3	9.05	0.54	0.24	0.58	(7.91, 10.19)	2.28
$DS = 0.58$	y_4	16.3	0.40	0.49	0.58	(13.98, 18.56)	4.58
	y_5	100	1.00	1.27	0.58	(94.01, 105.99)	11.98

- Candioti et al. (2006) pointed out that y2<1 is not acceptable.
- There is a much smaller chance that the actual response of y2 at our optimal solution is below 1 using the augmented approach.

Related techniques

- Kim and Lin (2000) provided a maximin formulation for the problem of correlated responses.
- Wu (2005) incorporated their correlations into the Harrington’s desirability function.
- Kros and Mastrangelo (2001) discussed additive desirability function (the arithmetic mean).
- Ortiz et al. (2004) and Mandal et al. (2007) combined a penalty function in Harrington’s desirability function approach to account for the constraints on the responses.

II. Balancing location and dispersion effects for multiple responses

Hsiu-Wen Chen, Hongquan Xu and Weng Kee Wong (2012)

Quality and Reliability Engineering International, in press.

Outline of Section II

- Why do we consider both location and dispersion effects?
- How to add both location and dispersion effects to the desirability function?
- Data application

Why do we consider both location and dispersion effects?

Robust design

- Fit a model that links the mean response with the input factors
- Assumption: all observations have equal variance
- Assumption may not always be valid
- Minimize the variation to achieve a robust process

Dual response surface methodology

- Replicates are available
- Step 1.
  primary response: location model (mean)
  secondary response: dispersion model (sd)
- Step 2.
  identify factor settings that simultaneously
  - optimize mean
  - minimize sd

Location and dispersion effects for multiple responses

- NTB problem: minimizing the variation
- LTB/STB problem: optimizing the mean response
- Balancing the relative importance of location and dispersion effects when there are multiple responses is an important and challenging task.

How to add both location and dispersion effects to the desirability function?

Location and dispersion models

Assume responses are independent.
Suppose there is a set of factor variables $\boldsymbol{x} = (x_1, x_2, \ldots, x_k)$

- The location model:

$$y_{\mu_i} = f_{\mu_i}(\boldsymbol{x}) + \varepsilon_{\mu_i}, i = 1, 2, \ldots, m,$$

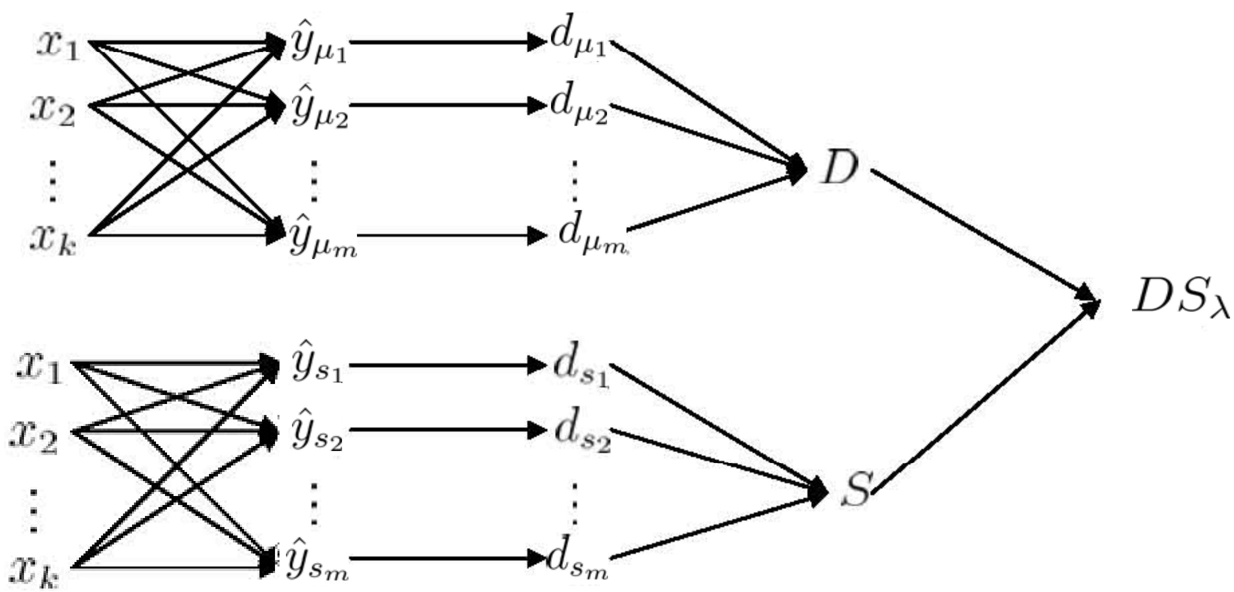
where y_{μ_i} is the sample mean over the replicates for ith response.

- The dispersion model:

$$y_{s_i} = f_{s_i}(\boldsymbol{x}) + \varepsilon_{s_i}, i = 1, 2, \ldots, m,$$

where y_{s_i} is the sample standard deviation over the replicates for ith response.

Proposed approach



$$DS_\lambda = D^\lambda S^{1-\lambda} = \left(d_{\mu_1} d_{\mu_2} \cdots d_{\mu_m}\right)^{\lambda/m} \left(d_{s_1} d_{s_2} \cdots d_{s_m}\right)^{(1-\lambda)/m}$$

Related techniques

- Maximin desirability approach (Kim and Lin, 2006). maximize the minimum of all location and dispersion desirability functions.
- Expected desirability approach (Lee and Kim, 2007). the average of the existing desirability values based on the probability distribution of the predicted response.

They typically provide a single solution that does not have the capacity to examine the role of location and dispersion effects simultaneously.

Data application

Colloidal gas aphrons experiment

- The data set is taken from Jauregi et al. (1997)
- Goal: find factor settings that optimize the location and dispersion effects of three responses
- Kim and Lin (2006) analyzed this data set using the maximin desirability function technique

Response		Type
y1	stability (log(seconds))	LTB
y2	volumetric ratio (none(ratio))	STB
y3	temperature (°C)	NTB
Factor		
x1	concentration of surfactant	
x2	concentration of salt	
x3	time of stirring	

- central composite design with 8 factorial points*, 6 axial points*, and a center point**

(* replicated twice, ** replicated 6 times)

- experimental region:

$$-1 \le x_k \le 1$$

$$k = 1, 2, 3$$

j	x_1	x_2	x_3	Replicate	y_{j1}	y_{j2}	y_{j3}	y_{μ_1}	y_{μ_2}	y_{μ_3}	y_{s_1}	y_{s_2}	y_{s_3}
1	-1	-1	-1	1	4.50	0.17	29.00						
	-1	-1	-1	2	4.50	0.26	23.00	4.50	0.22	26.00	0	0.06	4.24
2	1	-1	-1	1	6.04	0.50	23.00						
	1	-1	-1	2	6.39	0.53	25.40	6.22	0.52	24.20	0.25	0.02	1.70
3	-1	1	-1	1	3.81	0.17	22.00						
	-1	1	-1	2	4.09	0.20	27.00	3.95	0.19	24.50	0.20	0.02	3.54
4	1	1	-1	1	5.67	0.44	25.50						
	1	1	-1	2	5.19	0.40	21.00	5.43	0.42	23.25	0.34	0.03	3.18
5	-1	-1	1	1	4.67	0.32	20.00						
	-1	-1	1	2	4.22	0.32	41.00	4.45	0.32	30.50	0.32	0	14.85
6	1	-1	1	1	6.73	0.57	35.50						
	1	-1	1	2	6.57	0.57	18.00	6.65	0.57	26.75	0.11	0	12.37
7	-1	1	1	1	3.40	0.12	43.00						
	-1	1	1	2	4.32	0.28	20.00	3.86	0.20	31.50	0.65	0.11	16.26
8	1	1	1	1	5.72	0.46	19.00						
	1	1	1	2	5.09	0.50	34.00	5.41	0.48	26.50	0.45	0.03	10.61
9	-1	0	0	1	4.09	0.27	36.00						
	-1	0	0	2	4.38	0.23	24.00	4.24	0.25	30.00	0.21	0.03	8.49
10	1	0	0	1	5.52	0.52	30.00						
	1	0	0	2	5.39	0.51	24.00	5.46	0.52	27.00	0.09	0.01	4.24
11	0	-1	0	1	5.92	0.61	32.00						
	0	-1	0	2	5.93	0.59	23.40	5.93	0.60	27.70	0.01	0.01	6.08
12	0	1	0	1	4.74	0.36	36.00						
	0	1	0	2	4.50	0.30	21.00	4.62	0.33	28.50	0.17	0.04	10.61
13	0	0	-1	1	5.01	0.36	27.00						
	0	0	-1	2	4.70	0.25	24.00	4.86	0.31	25.50	0.22	0.08	2.12
14	0	0	1	1	4.94	0.53	38.00						
	0	0	1	2	5.01	0.51	25.00	4.98	0.52	31.50	0.05	0.01	9.19
15	0	0	0	1	4.85	0.47	34.00						
	0	0	0	2	4.94	0.46	34.00						
	0	0	0	3	4.98	0.49	33.00						
	0	0	0	4	4.89	0.48	24.00						
	0	0	0	5	4.94	0.46	19.00						
	0	0	0	6	5.01	0.47	25.00	4.94	0.47	28.17	0.06	0.01	6.37

Location and dispersion models

Following Kim and Lin (2006)

- Location models

$$\hat{y}_{\mu_1} = 4.95 + 0.82x_1 - 0.45x_2 - 0.15x_1^2 + 0.28x_2^2 - 0.11x_1x_2 + 0.07x_1x_3,$$

$$\hat{y}_{\mu_2} = 0.46 + 0.13x_1 - 0.06x_2 + 0.05x_3 - 0.07x_1^2 - 0.04x_3^2,$$

$$\hat{y}_{\mu_3} = 28.36 - 1.48x_1 + 2.33x_3 - 0.15x_1^2 - 1.42x_2^2 - 0.71x_1x_3,$$

- Dispersion models

$$\hat{y}_{s_1} = 0.06 + 0.11x_2 + 0.06x_3 + 0.12x_1^2 + 0.11x_3^2 - 0.1x_1x_3 + 0.05x_2x_3,$$

$$\hat{y}_{s_2} = 0.02 - 0.01x_1 + 0.01x_2 - 0.01x_3 + 0.02x_3^2 - 0.01x_1x_3 + 0.02x_2x_3,$$

$$\hat{y}_{s_3} = 6.08 - 1.53x_1 + 0.5x_2 + 4.85x_3 + 2.26x_2^2 - 0.65x_1x_3 - 0.67x_1x_2x_3,$$

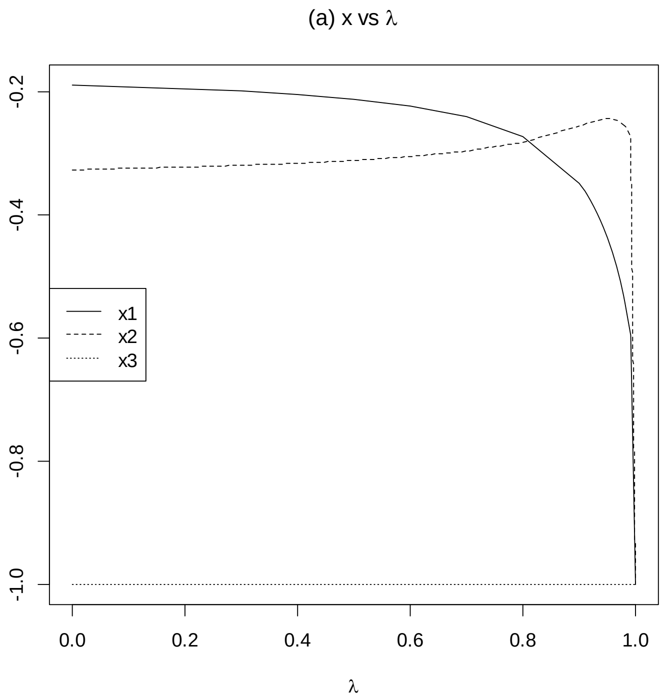
Transformation

Following Kim and Lin (2006)

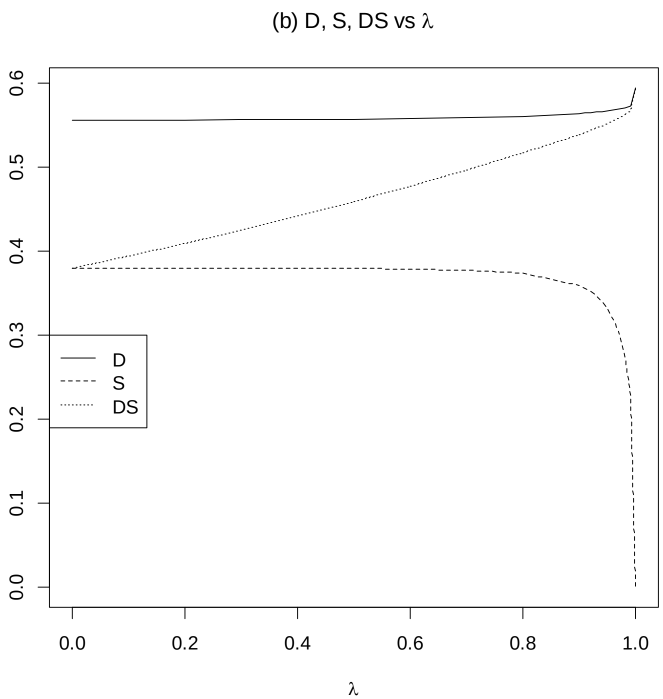
- The bounds and the target value of the location and dispersion effects for each response are given in

Responses	y_1	y_2	y_3
Type	LTB	STB	NTB
($y_{\mu_i}^{min}$, $y_{\mu_i}^{max}$)	(3.0, 7.0)	(0.1, 0.6)	(15, 45)
T_{μ_i}			30
($y_{s_i}^{min}$, $y_{s_i}^{max}$)	(0, 0.1)	(0, 0.1)	(1, 2)

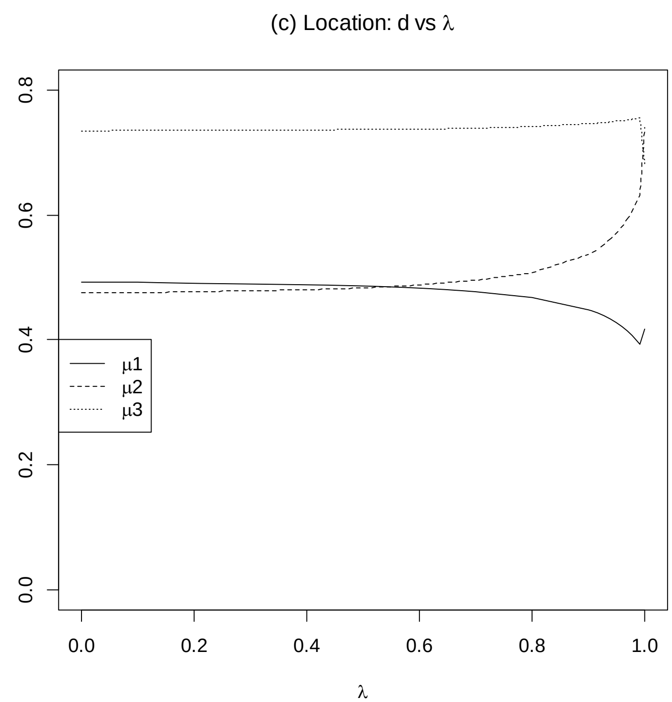
- All shape parameters are set at 1

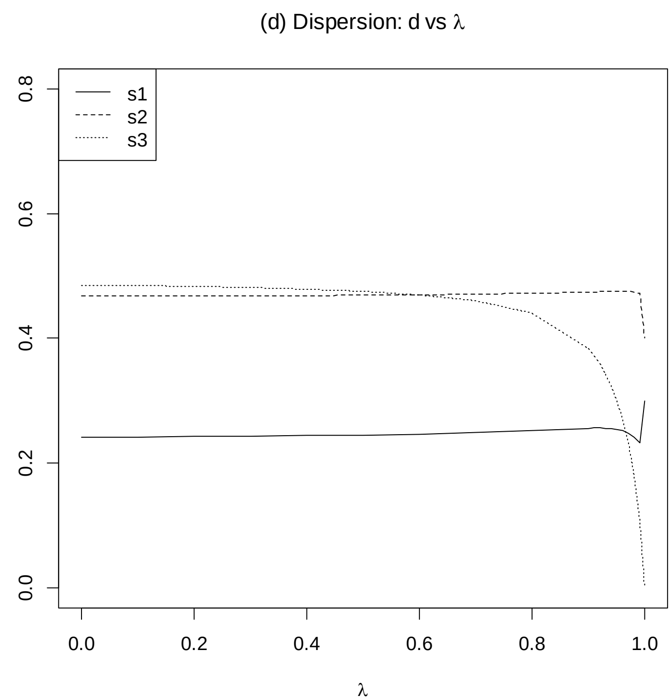


(a) x1 and x2 are quite sensitive to λ, implying that x1 and x2 are important for the location and dispersion effects;
optimal x3 is always located at -1, which suggests that the minimum time of stirring is optimal.



(b) S=0 when λ=1, implying that some response has an unacceptable variation;
S>0 when λ<1, indicating that the proposed approach is minimizing the overall dispersion effect;
The values of D are relatively insensitive to λ when λ<1.





(c) d_{μ_1} and d_{μ_2} change in conflicting directions when λ>0.5;
d_{μ_3} does not vary much as the value of λ changes.

(d) d_{s_3} decreases rapidly when λ>0.6;
d_{s_1} and d_{s_2} almost remain the same when λ<0.9.

Because y1 is the LTB type, y2 is the STB type, and y3 is the NTB type, we emphasize more on d_{μ_1}, d_{μ_2}, d_{s_3} than d_{s_1}, d_{s_2}, d_{μ_3}.

Any choice of 0.5<λ<0.9 gives a reasonable compromise solution that balances multiple location and dispersion effects.

Optimization results				RTY	MCpk
Conventional approach	x_* = (-1.00, -1.00, -1.00)			98.7873%	2.31
($\hat{y}_{\mu_i}$, d_{μ_i})	(4.67, 0.42)	(0.23, 0.74)	(25.23, 0.68)		
($\hat{y}_{s_i}$, d_{s_i})	(0.07, 0.30)	(0.06, 0.40)	(4.54, 0)		
D=0.60, S=0					
Maximin approach	x_* = (-0.21, -0.40, -1.00)			99.9994%	3.14
($\hat{y}_{\mu_i}$, d_{μ_i})	(5.00, 0.50)	(0.36, 0.47)	(25.96, 0.73)		
($\hat{y}_{s_i}$, d_{s_i})	(0.07, 0.30)	(0.05, 0.46)	(1.63, 0.37)		
D=0.56, S=0.37					
Expected approach	x_* = (-0.94, -0.88, -1.00)			99.6430%	2.43
($\hat{y}_{\mu_i}$, d_{μ_i})	(4.63, 0.41)	(0.24, 0.72)	(25.53, 0.70)		
($\hat{y}_{s_i}$, d_{s_i})	(0.07, 0.30)	(0.06, 0.41)	(3.91, 0)		
D=0.59, S=0					
Proposed approach:					
$\lambda = 0.5$	x_* = (-0.21, -0.31, -1.00)			99.9997%	3.16
($\hat{y}_{\mu_i}$, d_{μ_i})	(4.95, 0.49)	(0.36, 0.48)	(26.05, 0.74)		
($\hat{y}_{s_i}$, d_{s_i})	(0.08, 0.24)	(0.05, 0.47)	(1.52, 0.48)		
D=0.56, S=0.38					
$\lambda = 0.9$	x_* = (-0.35, -0.26, -1.00)			99.9999%	3.16
($\hat{y}_{\mu_i}$, d_{μ_i})	(4.80, 0.45)	(0.33, 0.54)	(26.19, 0.75)		
($\hat{y}_{s_i}$, d_{s_i})	(0.07, 0.26)	(0.05, 0.47)	(1.62, 0.38)		
D=0.56, S=0.36					

- Conventional desirability approach (Harrington, 1965) does not consider the dispersion effects
- Expected desirability approach (Lee and Kim, 2007) also leads to an unacceptable solution even when the dispersion effects are considered

• Proposed approach provides multiple optimal solutions.

• Our optimal solutions (at both $\lambda=0.5$ and $\lambda=0.9$) have the highest rolled throughput yield (RTY) and overall multiple capability (MCpk) values among all four approaches.

(Assume a normal distribution for all the responses.

- Rolled throughput yield (RTY): probability that all responses simultaneously meet their respective bounds.

- Overall multiple capability (MCpk) is defined as the geometric mean of the individual capability indices Cpk's of all the responses.

• Large RTY and MCpk values are preferred.)

Summary

- Multiple response optimization: find optimum conditions on x that balance multiple responses y
- The desirability function can be particularly beneficial when many responses are involved in the optimization
- Our augmented desirability function offers multiple compromise solutions and greater flexibility because it can incorporate various types of information or optimality criteria with the relative weight
  - Section I: minimize prediction variances
  - Section II: balance location and dispersion effects
- The estimated optimum conditions from one experiment is just that -- an estimate
- Reveal important information about the process and information about the role of the variables

  ex: how different levels of x affect each of the responses

Thank you

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