應用機率模型作業 2 解答

22.

By the independent of trail, after two points, there are three cases:

(i) A wins with probability p^2.

(ii) B wins with probability $(1-p)^2$.

(iii) Turn to the origin with $2p(1-p)$.

(a)

The game end at 2n points if it turn to origin after 2(n-1) points, and then A or B wins at the next 2 points.

$\therefore P(\text{the game end at 2n points})$

$=[2p(1-p)]^{n-1}\cdot[p^2+(1-p)^2]$

(b)

$P(\text{A wins})$

$=P(\text{A wins at 2 points})+P(\text{A wins at 4 points})+\dots$

$=p^2+[2p(1-p)]\cdot p^2+[2p(1-p)]^2\cdot p^2+\dots$

$$=\frac{p^2}{1-[2p(1-p)]}=\frac{p^2}{1-2p+2p^2}$$

41.

We could know that their parent are both (A,a).
Note that 'A' is black gene, and 'a' is brown one.

(a)

$$P(\text{Pure black}|\text{black})$$
$$=\frac{P((\text{Pure black})\cap(\text{black}))}{P(\text{black})}$$
$$=\frac{1/4}{3/4}=\frac{1}{3}$$

(b)

E: Pure black, E^c: not pure black
F: five offsprings black

$$P(E|F)$$
$$=\frac{P(E\cap F)}{P(F)}$$
$$=\frac{P(F|E)\cdot P(E)}{P(F|E)\cdot P(E)+P(F|E^c)\cdot P(E^c)}$$
$$=\frac{1\cdot\frac{1}{3}}{1\cdot\frac{1}{3}+\left(\frac{1}{2}\right)^5\cdot\frac{2}{3}}=\frac{16}{17}$$

75.

(a)

Knowing that values of $\{N_1, N_2, ..., N_n\}$ is equivalent to knowing the relative ordering of the elements $\{a_1, ..., a_n\}$.

For instance, if $N_1 = 0, N_2 = 1, N_3 = 1,$ then in the random permutation a_2 is before a_3, which is before a_1.

The independence result follows for clearly the number of $a_1, ..., a_i$ that follow a_{i+1} does not probabilistically depend on the relative ordering of $a_1, ..., a_i$.

(b)

$P(N_i = k)$

$= P($there are k units in $a_1, a_2, ..., a_{i-1}$ proceed $a_i)$

$$= \frac{C_k^{i-1} \cdot k! \cdot [(i-1)-k]! \cdot \left[(i+1)\cdot(i+2)\cdot ... \cdot(n-1)\cdot n\right]}{n!} = \frac{1}{i}$$

(c)

$$E(N_i) = \frac{i-1}{2}$$

$$\therefore E(N) = E(\sum_{i=1}^{n} N_i) = \frac{n\cdot(n-1)}{4}$$

$$Var(N_i) = E(N_i^2) - [E(N_i)]^2$$

$$= \frac{(i-1)(2i-1)}{6} - \left[\frac{(i-1)}{2}\right]^2 = \frac{i^2-1}{12}$$

$$\therefore Var(N) = Var(\sum_{i=1}^{n} N_i) \overset{id}{=} \sum_{i=1}^{n} Var(N_i)$$

$$= \frac{1}{12}\left[\frac{1}{6}n(n+1)(2n+1) - n\right]$$

$$= \frac{2n^3 + 3n^2 - 5n}{72}$$