應用機率模型作業 4 解答

P175 Question21, P177 Question 31, P179 Question 40

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(a)

$$X=\sum_{i=1}^{N}T_i$$

(b)

$$N\sim Geo(\frac{1}{3}),E(N)=3.$$

(c)

$$E(T_N)=2.$$

(d)

$$E(\sum_{i=1}^{N}T_i\mid N=n)=E(\sum_{i=1}^{N-1}T_i+T_N\mid N=n)=E(\sum_{i=1}^{n-1}T_i)+E(T_n)=(n-1)(\frac{3+5}{2})+2=4(n-1)+2.$$

(e)

$$E(X)=EE(\sum_{i=1}^{N}T_i\mid N)=E(4(N-1)+2)=10.$$

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(a)

$Let\ R_1:the\ length\ of\ the\ first\ run$

$$X=\begin{cases}1;if\ the\ first\ outcome\ is\ 1\\0;o.w.\end{cases}$$

$$E(R_1)=EE(R_1|X)=E(R_1|X=1)P(X=1)+E(R_1|X=0)P(X=0)$$

$$=(\frac{1}{1\text{-}p})p+(\frac{1}{p})(1\text{-}p).$$

(b)

$Let\ R_2:the\ length\ of\ the\ \sec ond\ run$

$$X=\begin{cases}1;if\ \ the\ \ first\ outcome\ is\ 1\\0;o.w.\end{cases}$$

$$E(R_2)=EE(R_2|X)=E(R_2|X=1)P(X=1)+E(R_2|X=0)P(X=0)$$

$$=(\frac{1}{1\text{-}p})(1-p)+(\frac{1}{p})p=2.$$

40

(*a*)

Let W : *the number of days until he reachs freedom*

$$Y = \begin{cases} 1; if \ \ choose \ \ door \ \ 1 \\ 2; if \ \ choose \ \ door \ \ 2 \\ 3; if \ \ choose \ \ door \ \ 3 \end{cases}$$

$$E(W) = EE(W \mid Y) = 0.5(2 + E(W)) + 0.3(3 + E(W)) + 0$$

$$\Rightarrow E(W) = 9.5(days).$$

(*b*)

Consider all possibility

$$[(2+3)(\frac{1}{3}\times\frac{1}{2}) + 2(\frac{1}{3}\times\frac{1}{2})] + [(3+2)(\frac{1}{3}\times\frac{1}{2}) + 3(\frac{1}{3}\times\frac{1}{2})] + [0(\frac{1}{3})] = \frac{5}{2}.$$

(*c*)

For part a

$$M_W(t) = EE(e^{wt} \mid Y) = 0.5E(e^{t(W+2)}) + 0.3E(e^{t(W+3)}) + 0.2E(e^{t(0)})$$

$$\Rightarrow M_W(t) = \frac{2}{10 - 5e^{2t} - 3e^{3t}}$$

$$\Rightarrow M_W^{"}(0) = E(W^2) = 204$$

$$\Rightarrow Var(W) = E(W^2) - (E(W))^2 = 204 - (9.5)^2 = \frac{455}{4}.$$

For part b

$$Var(W) = [(2+3)^2(\frac{1}{3}\times\frac{1}{2}) + 2^2(\frac{1}{3}\times\frac{1}{2})] + [(3+2)^2(\frac{1}{3}\times\frac{1}{2}) + 3^2(\frac{1}{3}\times\frac{1}{2})] - (\frac{5}{2})^2 = \frac{17}{4}.$$